



HAL
open science

A Macrodynamic Model of Kant's Theory of War and Democracy

Laurent Gauthier, Antoine Parent, Vincent Touzé

► **To cite this version:**

Laurent Gauthier, Antoine Parent, Vincent Touzé. A Macrodynamic Model of Kant's Theory of War and Democracy. 2024. hal-04390576v1

HAL Id: hal-04390576

<https://univ-paris8.hal.science/hal-04390576v1>

Preprint submitted on 12 Jan 2024 (v1), last revised 17 Jan 2024 (v2)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

A Macrodynamic Model of Kant's Theory of War and Democracy*

Laurent Gauthier[†] Antoine Parent[‡] Vincent Touzé[§]

January 12, 2024

Abstract

We propose a macrodynamic model of Kant's theory of the relationship between war and democracy. War and democracy have always been linked in some ways, from Athen's democratic empire in the 5th Century BCE, through the industrialization of war in the 19th and 20th centuries to today's nuclear powers, all forms of political regimes have been involved in war. Various macroeconomic models have accounted for the costs and benefits of war, and we extend them by introducing regime-specific sensitivity to human loss, which affects the desirability of war. We show the model can account for multiples wars throughout history. **JEL Codes:** E22, F5, H56, O41, N4. **Keywords:** War, history of war, democracy, one-sector growth model.

*We wish to thank participants at the 2021 Conference on Complex Systems in Lyon, and participants at the 2022 and 2023 conferences "Understanding Revolutions, War and Peace: A View From Cliometrics and Complexity" at the University of Chicago in Paris. We also extend our thanks to Patrice Abry, Cécile Bastidon, Francis Bloch, Pierre Borgnat, Steven Durlauf, Oded Galor, Mikhail Golosov, Antoine Le Riche, László Lovász and Maxime Menuet.

[†]laurent.o.gauthier@gmail.com, LED, Université Paris 8, CAC IXXI, ENS Lyon, ESPRI/ArScAn, Université Paris Nanterre.

[‡]antoine.parent02@univ-paris8.fr, LED, Université Paris 8, CAC IXXI, ENS Lyon and OFCE – Sciences Po.

[§]vincent.touze@sciencespo.fr, OFCE – Sciences Po.

In his essay *Zum ewigen Frieden: ein philosophischer Entwurf*, Kant ([1795] 1991) deployed his analysis of perpetual peace and, in particular, he theorized that democratic states would not be war-prone. This perspective has evolved into a form of conventional wisdom, and has been extensively studied, both theoretically and empirically, in the fields of political sciences and international relations. In this paper, we develop a macroeconomic model that can account for Kant's theory, and discuss its application on historical data.

We effectively develop a theory of conflict and war based on economic power relations. We consider two countries whose competition is expressed solely by their ability to wage war against each other, depending on the evolution of their respective productive capacities. The evolution of productive capacity is based on a model of capital accumulation *à la* Solow (1956). Sovereign choices about whether or not to wage war are based on trade-offs between economic gains and costs. The evolution of the balance of economic power is endogenous to the capitalist dynamics of each of the two economies. Independently, different transitional or permanent regimes of peace, unilateral war or total war can be realized. Then, relying on the idea that democracy is the outcome of the diffusion of humanist thought, we show that, in a world without an international force capable of protecting each country against the aggression of another, taking into account the value of each human life lost, on a national or universal level, affects the sovereign decision process, thereby creating conditions more conducive to the emergence of peace. Avoiding a binary opposition between democracy and autocracy, we consider a continuum of different types of democratic and authoritarian regimes, which is effectively justified by several existing classifications of political regimes in the literature. Indeed, we propose a specification that accounts for the hierarchy and differential values allocated to the destruction of capital and human lives in the decision process of belligerents' countries, following the typologies of democratic and autocratic regimes borrowed from Töngür, Hsu, & Elveren (2015) and Vanhanen (2000), in particular.

Our arguments are organized as follows. In the first section, we discuss the existing research in various fields, ranging from political science and sociology to history and economics, which has addressed the existence and emergence of democracy, in connection with war. In the second section, we model Kant's proposition: we begin by defining a macroeconomic model for two economies at peace or at war, in a comparable fashion to Lagerlöf (2010). We then consider that the nature of a political regime, in its relation with war, can be efficiently captured through a simple metric measuring its aversion to human loss. Applying this logic, we show how the instantaneous decision to go to war, as an optimal control, is affected by the political regime. Finally, we show in the third section how the model can account for a variety of conflict situations drawn from history.

1 Democracy, War and Economics

Since the first systematic studies of war by Quincy Wright, informed by the then unpublished work of Lewis Richardson on the statistics of war (Richardson [1960] 1975), and relying on the entire gamut of hard sciences and social sciences (Wright 1942a, 1942b), the science of war, looking to understand the phenomenon under all relevant perspectives, and in particular in relation with political regimes, has made significant progress. These initial efforts were seminal in the later founding of research outlets such as the *Journal of Conflict Resolution* and the *Journal of Peace Research* in the late 1950s and early 1960s (Freedman 2018, 111–112). Putting together exploitable datasets on war and on democracy has since been a significant effort, and it has allowed for the emergence of empirical-driven research into the causes of war.

In this first section, we first survey research in political science and sociology that has addressed the question of war and its relationship with democracy. We also discuss some approaches that have put into question the notion that democracies may be reluctant to go to war. We then turn to macroeconomic approaches to war and democracy.

1.1 The Democratic Peace Proposition in Political Science

Democracies do not go to war: Babst (1972) found no war between independent nations with elective governments from 1789 to 1941 (p. 55). Democratic regimes preclude violence (Rummel 1975–1981, 279). Levy contends that the “democratic peace proposition” (the absence of wars between democracies) is “as close as anything we have to an empirical law in international relations” (Levy 1988, 662). Gleditsch found “perfect” correlation between democracy and the absence of war (N. P. Gleditsch 1992, p 372). Possible exceptions to this rule have been identified and discussed (Ray 1993, 1998), including the War of 1812 between the United States and Great Britain, the American Civil War, the Boer War, the Spanish-American War, the First World War, however these wars involved at least one autocratic regime. Other sources of controversy focuses on the definition of democracy. If democracy is viewed as a time-dependent concept (Oren 1995), then the validity of the democratic peace proposition is impossible to evaluate. This argument may seem specious in that it denies the universal principles of democracy.

Most empirical tests evaluating the democratic peace proposition have been realized using data generated by Gurr et al. (Gurr 1974; Gurr 1978; Gurr, Jagers, & Moore 1990; Jagers & Gurr 1995); these studies used the dataset *Polity III*. Another comparable dataset that has been used in this context is the *Correlates of War* data (Carter, Wolford, & Bennett 2022),

a project initiated by Singer (1972), and which has benefited from further improvements, such as those proposed by K. S. Gleditsch & Ward (1999) or by Gibler, Miller, & Little (2016) (the *Militarized Interstate Dispute* data). Many distinct research efforts have attempted to categorize political regimes. Töngür, Hsu, & Elveren (2015), for example, distinguish social democracy, conservative democracy, one-party democracy, dictatorship, military dictatorship, civil war, and communist regime. The *Polity* database uses political participation, government recruitment, and degree of democratic constraint on the chief executive. Other commonly used continua measures include Freedom House (2022) and Vanhanen (2000). Freedom House ranks degree of democracy by examining election outcomes and balance of power. Vanhanen measures degree of democracy using percentage votes for smaller parties and percentage of adults voting in elections. Hadenius & Teorell (2007) use the Polity/Freedom House database to sub-classify regimes in terms of hereditary succession, use of military force, and the presence or absence of popular elections into twenty types of sub-regimes.

Ray (1998) surveys the limitations of empirical evaluation. Up to the current period, the share of democratic states in the world is scarce: the democratic peace proposition may reflect this statistical artifact. Maoz & Abdolali (1989), using the data from Gurr (1974) and Gurr (1978), recorded every pair of states in the world from 1816 to 1976, for a total of 271,904 observations. Over this period of 160 years, Maoz & Abdolali (1989) found that “democratic states never fight one another” (p. 21). Bremer (1993) observing the very few democratic states in the pre-World War II era (1816–1939) questions the utility of including these years in the sample. Farber & Gowa (1995) and Farber & Gowa (1997) underline that alliance ties between democracies (such as NATO), because of the common opposition of their members to communism, may impede democracies to fight against other democracies. Bremer (1993) concludes that “even after controlling for a large number of factors democracy’s conflict reducing effect remains strong” (p. 246). Globally, empirical analyses support the democratic peace proposition.

Theoretical bases for the democratic peace proposition can be found in Rummel ([1997] 2017) for whom democracy is “an important pacifying force”, notably through the role of opinion and electoral processes. Bueno de Mesquita & Lalman (1992) develop a game-theoretical analysis of international interactions that underlines the role of domestic political incentives on bargaining among democratic states. Conversely to autocratic regimes, institutional, cultural and normative constraints in democracies favor peaceful resolutions of conflicts and influence the preference for bargaining between democratic states. For Oneal & Ray (1997), international trade among democratic states has a complementary pacifying impact. Russett (1998) finds that the existence of international institutions contribute to peaceful resolution of conflicts between states. Dixon (1993), Dixon (1994) and Raymond (1994) report that

democratic states are more inclined to use mediation and arbitration, than autocratic regimes. Lastly, Ray (1998) mentions the issue of democratic reelection as a pro-peace mechanism in democracies (p. 40): leaders in democracies may be reluctant to trigger wars against other democracies because this could harm their chances of reelection. Ray suggests political leaders in democracies avoid wars against other democratic states for “strategic, logical or self-interested reasons, e.g. the impact of lost wars on their personal political fates”.

1.2 Politics by Other Means

A more cynical view on the relationship between war and democracy is to consider it as a simple diversion, and historians and political scientists have studied the diversionary motive in international conflicts. This reflects von Clausewit’s well-known dictum that war is merely the extension of politics by other means (Clausewitz [1832] 1989). Mansfield & Snyder (1995) discuss and cite examples of diversionary conflicts of “democratizing regimes” of Victorian Britain, the France of Napoleon III, Bismarck and Wilhelmine Germany: in each of these cases, they point out, “elections were being held and political leaders were paying close attention to public opinion in the making of foreign policy” (p. 316). However, these regimes were not strongly democratic. Mayer (1969) provides an analysis of the domestic political influence on external conflicts in Europe during the late 19th and mid 20th centuries: “precisely because their internal influence and control are tenuous, actors and classes are incline to have recourse to external war which, if successful, promises to shore up their faltering positions” (p. 294).

The particular case of civil war can also be perceived as a form of political engagement. N. P. Gleditsch & Hegre (1997) and Hegre (2014) report that very few studies find traces of a monotonic effect of democracy on civil war and that a number of studies find empirical confirmation of an “inverted-U” relationship between level of democracy and civil war. Semi-democratic regimes have a higher risk of internal conflict than pure autocracies or democracies. The existence of this “inverted-U” has been challenged, by Elbadawi & Sambanis (2002), Collier & Hoeffler (2002), Vreeland (2008). When controlling for GDP per capita or other indicators of socio-economic development, democracies have no lower risk of internal armed conflict than autocratic regimes. Nonetheless, Lacina (2006) and N. P. Gleditsch, Hegre, & Strand (2009) find that civil wars in democracies are less lethal. Democratic governments make use of less violence against civilians (Eck & Hultman 2007) and engage in less repression (Davenport 2007; Colaresi & Carey 2008), but rebel groups tend to use more violence against civilians when fighting democratic regimes (Eck & Hultman 2007). Democracies tend to have longer internal wars (N. P. Gleditsch, Hegre, & Strand 2009). One aspect of the effectiveness of democracies in war is their ability to form large alliances (Doyle 1986; Raknerud & Hegre

1997). Democratic institutions are the solution to this commitment problem (Acemoglu & Robinson 2005). This would explain why democratic institutions reduce the risk of (revolutionary) civil wars. Civil war can also be envisioned at the core of a democratic process: R. K. Fleck & Hanssen (2013), for example, have shown that in many instances the first democratic transitions, in ancient Greece, were consequences of tyranny which had the effect of uniting citizens against the autocrat, hence accounting for one side of the relationship. Robert K. Fleck & Hanssen (2006) also proposed, and empirically tested, a model for the emergence of democracy in ancient Greece, where the ruling oligarchy had an interest in giving rights to the citizens and effectively give away the power to predate on them, avoiding civil war, in order to promote long term personal investment and improved economic growth. Another perspective that may account for the existence of the “inverted-U” relationship is the presence of bias in decisions, as examined by Jackson & Morelli (2007): elected rulers may have different risk and reward perceptions from the whole country, leading to a greater or smaller occurrence of war than would otherwise be anticipated. Finally, it is important to stress that carrying out analysis on civil violence through long periods of time, especially if one considers Late Medieval or Early Modern times, raises all manners of historiographical issues, because of the nature of the data and the difficulties in its interpretation (Carroll 2017).

The link between democracy and peace may indeed not be as systematic as one may believe *a priori*. Two main objections have been raised to the causal inference “democracy causes peace”. Thompson (1996) puts forward that geopolitical constraints before democratization can explain subsequent peace. For Gates, Knutsen, & Moses (1996), peace leads to trade, investment, and economic growth, and thereby to democratization. Gibler (2007) and Boix (2003) note the importance of the settlements of territorial claims in 17th- and 18th-century Europe that induced the fundamental economic changes for democratization to occur. Such territorial agreements, they contend, led to “clusters of democracies” that share a similar interest in preserving peace. Nevertheless, Goenner (2004) has shown using careful variable selection that “trade interdependence does not have a significant effect on the prediction of militarized conflict”. Hegre (2014) emphasizes that a third factor, namely socioeconomic development, drives both democratization and peace. He suggests an institutional explanation *à la* North, according to which both democracy and peace are the outcomes of deeper and former societal changes. In the same vein, Mousseau (2000) defends the view that market norms and the habit of contracting actually facilitated both democratic institutions and the democratic practice of peace. These norms emerge in economically developed countries by a “process of cultural materialism”. Wars of conquest would violate these norms. Rosecrance (1986) argues that commerce replaced conquest “since labor, capital, and information are mobile and cannot be definitively seized” (p. 48). In this view, the democratic peace is

assessed as a function of economic development, a perspective also developed in Gartzke (2007). Lastly, in many sociological theories of democratization (Dahl [1971] 2007; Olson 1993; Boix 2003), pluralist societies are seen as an essential prerequisite for democracy. Another perspective is the impact of war on economic growth, perceived as a positive, which has recently been addressed by Sevastianova (2009) and Thies & Baum (2020). The opposite proposition, that democracy may cause war, has also been examined in the case of ancient Athens: in a political structure where the citizens who actually stand to individually benefit from war are the ones who decide to engage in war, it is logical to observe a strong propensity towards armed conflict, as was the case for the city-state in the 5th century BCE (Tridimas 2015).

1.3 The Macroeconomic Theories of War and Peace

In spite of its paramount importance in macroeconomics, the occurrence of war has not been widely examined from a macrodynamic perspective, and in particular in its relation with democracy. To the best of our knowledge, Hess & Orphanides (2001) present the very first general equilibrium model of conflict to investigate “whether the prevalence of democracy is sufficient to foster the perpetual peace hypothesized by in Kant ([1795] 1991) and whether the world would necessarily become more peaceful as more countries adopt democratic institutions”. They justify their exclusively theoretical approach to this issue, arguing that strictly empirical studies of the “democratic peace proposition” face sample size and sample selection problems. Hess & Orphanides (2001) build a model in which “the equilibrium frequency of war is endogenously determined by the relative prevalence of democratic and non democratic regimes, the relative importance of an appropriate motive for war, and the influence of a war’s outcome to affect an incumbent leader’s ability to hold on power”. The contribution of this approach is that it introduces different sub-cases of democracy and authoritarian regimes (“partial benevolence” / “selfishness” / “an idealistic Kant equilibrium referred as to a Pareto optimum” / “appropriative wars” / “welfare improving wars”) and discusses the implications for peace according to these different sub-cases. The nuanced conclusion of their general equilibrium model of conflict is that, depending on these sub-cases, “democracy can coexist with war but maintaining a democratic perpetual peace will be most likely only with increased international integration and coordination”.

Lagerlöf (2010) develops a two-country two-sector long-run overlapping generations growth model *à la* Hansen & Prescott (2002) introducing war by letting the countries take land from each other, at the cost of destroying capital and killing people. The opportunity to wage war is given to each country sequentially. Firstly, Lagerlöf notes that “the decline in

Great Power warfare came together with a rise in per capita incomes in Western Europe.” Thus, he suggests “a connection between warfare and economic development among Great Power nations” (p. 619). His underlying idea is that the economy eases tensions and turns countries away from war. Secondly, Lagerlöf aims at building a model that is consistent with the elimination of Great Power warfare in the 19th century, when most Great Powers industrialized. He provides an explanation of the secular decrease in Great Power wars in the 19th century by the economic transition of western economies from stagnation (the Malthusian equilibrium) to growth (the Solow equilibrium). In a manner consistent with Galor & Weil (2000), productivity grows at an exogenous rate that is faster in the Solow sector than the Malthus sector, generating a transition to a Solow economy at some point. Takeoffs from stagnation to growth and the arbitrage between predation and production derives from the transition from a Malthusian to a Solow equilibrium.

This interesting work is not without its own limitations: how does it deal with the resurgence of Great Power conflicts associated with the two World Wars during the 20th century? In his conclusion Lagerlöf recognizes that his model cannot explain 20th century warfare after the rise of the Solow sector. One possibility, he suggests, is that the two world wars were caused by competition for other resources than land (coal, petroleum, metals). One can find the answer disappointing because, whatever the kind of war, land conquest or other, war in any case implies killing masses and destroying capital. Indeed, it is irrelevant to have a macrodynamics model that fits the 19th century and not the 20th or 21st century. If we call upon macrodynamics, it is for its principle of generality, not to obtain one model by period, which would then correspond to the outcome of a partial equilibrium model. Our purpose is to be more general and better specify our macrodynamic modeling by distinguishing two key factors relating to the destruction of capital and human lives in the utility function of the belligerent countries.

In another direction, Le Fur & Wasmer (2023) propose an extension to the Unified Growth Theory by introducing conflict over the allocation of land resources between two countries, combined with endogenous public choice of military spending and endogenous private choice of fertility and child education. They give a variety of applications to historical episodes of colonialism. Also focusing more specifically on wars aimed at obtaining resources, Acemoglu et al. (2012) have developed a macrodynamic model where countries wage war over a particular resource. As each country's own existing resources are depleted, they incur a greater incentive to go to war, which is also modulated by the other country's strength. They illustrate their model, *inter alia*, with the case of the late 19th century War of the Pacific between Chile, Peru and Bolivia, over a valuable resource: nitrates and guano. While this model does not

include an impact for the political regime for the belligerents, it clearly connects economic resources to the occurrence of war.

How do wars make states? Alesina, Reich, & Riboni (2020) address this specific question through the emergence of a nationalist sentiment. They develop a theory of territorial conflict between two economies where the world geography is mathematically summarized as a linear segment, the border being a point on that segment, positioned based on an endogenous balance of power. Both autocratic societies consist of two types of agents: the elite and the citizens who become soldiers when the elite decides to go to war. The probability of victory then depends on the effort of the soldiers. To control this effort, the elite use their ability to collect taxes, appropriate political rent, provide a public good, pay soldiers, and to unite people around communal values, including nationalism. There is no cumulative effect nor dynamics in this model. This theory provides a useful analytical and qualitative framework for understanding the forces that determine the fate of nations. The authors then propose an extensive discussion to show how their model can be linked to numerous historical facts and how some military decisions could be explained by specific national trade-offs.

Research approaches to war and democracy have hence relied on a wide variety of techniques, whether theoretical or empirical. Nevertheless, no model has been offered to specifically account for Kant's observation. In the next section, we develop a model in several stages, to account for the links between political regime and war.

2 Modeling Kant's Proposition on War and Democracy

The macroeconomic models developed by Hess & Orphanides (2001) and by Lagerlöf (2010) accounted for effects such as households' decisions, leader's discretionary decision to go to war, or multiple production sectors in the economy. For our purposes, we will consider a simpler setup, with a single sector, and simple dynamics, in order to more precisely focus on the mechanics of war: indeed, introducing the possibility of war makes the analysis more difficult, and in order to keep tractable results it is necessary for the underlying model to remain sufficiently straightforward.

In the next subsection, we define a macroeconomic model that can reflect the impact of war between two countries, a war that may be unilateral or bilateral. We explore the steady-state equilibria and the joint dynamics of both countries depending on whether they are at war with each other. This first level of modeling does not factor in the decision to go to war or not, but is concerned with the impact of war on the economy. Next, we extend the model to include the decision to go to war, and we examine the extent to which the political regime of

a country affects its assessment of the trade-off of going to war, depending on its aversion to the human loss of life associated to being at war. With this approach, we can categorize political regimes in terms of different thresholds in their decision to enter war, and we explore the resulting macrodynamics. Autocracies, in the model, are incited to wage war in many more situations than democracies.

2.1 Macrodynamic Model for War and Peace

We consider two countries indexed by $i \in \{1, 2\}$ (we will write $-i$ to designate the other country, given a country i), characterized by a dynamics of capitalism *à la* Solow (1956). Following Solow, we focus on the stock of capital relative to production and express its variation in continuous time as a function of a savings rate $s < 1$, a production function f , and the natural rate of growth $\delta > 0$. We do not factor in economic interaction between the two countries in peace time, since, as we pointed out earlier based on the analysis by Goenner (2004), they do not appear to play a role in militarized conflict. In time of peace, each country is hence considered a closed economy.

First consider a discrete model, with the following dynamics for the capital stock per person: $k_{t+1} = sf(k_t) + (1 - \delta)k_t$, and the total production is $Y_t = Nf(k_t)$, with N the total population, which is assumed to be constant. In this model, each time-step represents a new generation of people, who work, have children, and pass on to their children a share of their production, according to the saving rate s . When each generation dies, the next one replaces it. In this framework, we will consider that if there happens to be war, then the human losses will affect the population (a given generation) after it has contributed to production, saved, and has had children. Therefore, human losses in war, in this setup, do not have a direct impact on the economy. As we will see, we will model the effect of war on the economy in a more explicit and direct fashion. Human losses in war will be accounted for as an exogenous input, specific to each country.

Since we examine long-term evolutions, we take the continuous limit of the model above, in order to improve tractability. In the simplest setting, with two countries, we obtain a version of Solow's growth model:

$$\begin{cases} \frac{dk_{1,t}}{dt} = s_1 f_1(k_{1,t}) - \delta_1 k_{1,t} \\ \frac{dk_{2,t}}{dt} = s_2 f_2(k_{2,t}) - \delta_2 k_{2,t} \end{cases}$$

Each equation corresponds to Equation 6 in Solow (1956). The function f is assumed to be continuous, increasing, concave, and bounded, with $f(0) = 0$. We also assume that f has

first and second derivatives almost everywhere. The population, who provides labor, is not explicitly modeled. This model is comparable to the model developed by Lagerlöf (2010), but expressed in continuous time, and without an explicit modeling of land scarcity and without an exogenous stochastic outcome from war.

In this context, we introduce the potential for war : between two countries, there may be no war, unilateral war if only one country is fighting but the other is not actively fighting back, or bilateral war if both countries are fighting. War induces both costs C and gains G from the perspective of the attacker, and a pure defensive cost D for each country when both are at war (all these gains and costs are expressed per unit of time). In addition, if there is war, while we do not assume that the population loss has a specific impact on production, we will write L_1 and L_2 the human loss due to war, the same whether the country is in a defensive or offensive position. This human loss measure can have an impact on the decision process towards war.

The offensive costs of war per unit of time for Country i attacking Country $-i$ writes: $C(Y_{i,t}, Y_{-i,t}) = P(Y_{-i,t}, Y_{i,t}) c(Y_{i,t})$, where P is the relative military power of the country under attack with respect to the attacking country: $P(Y_{-i,t}, Y_{i,t}) = \frac{Y_{-i,t}}{Y_{i,t}}$; and c the cost of attack, assumed to be proportional to domestic income, with a coefficient $\rho < 1$ specific to each country: $c(Y_{i,t}) = \rho_i Y_{i,t}$. The total cost is hence:

$$C(Y_{i,t}, Y_{-i,t}) = \rho_i Y_{-i,t} = \rho_i N_{-i} f_{-i}(k_{-i,t}).$$

This formulation expresses the fact that one's capability to wage war is reduced by the adversary's capabilities¹.

The gains of war expressed in terms of capital stock per person, from the economic predation of Country $-i$ by Country i , writes: $G(Y_{i,t}, Y_{-i,t}) = P(Y_{i,t}, Y_{-i,t}) g(Y_{-i,t})$, where the relative economic power of the domestic country with respect to the foreign country is $P(Y_{i,t}, Y_{-i,t}) = \frac{Y_{i,t}}{Y_{-i,t}}$; and g is the gain per unit of military power, with a parameter $\gamma < 1$, specific to each country, $g(Y_{-i,t}) = \gamma_i Y_{-i,t}$. The total gain is hence:

$$G(Y_{i,t}, Y_{-i,t}) = \gamma_i Y_{i,t} = \gamma_i N_i f_i(k_{i,t}),$$

¹This perspective recoups with the Lanchester directed-fire combat model (Lanchester 1916, 1956), and well-known and used in combat modeling (Washburn & Kress 2009; Kress 2020). Following the simple presentation in Niall MacKay (2006), a combat model can be expressed as a system as a function of R , the ability to do damage, where m is each side's effectiveness, with a form that resembles our expression for the cost of war: $\left\{ \frac{dR_1}{dt} = -m_2 R_2, \frac{dR_2}{dt} = -m_1 R_1 \right\}$. The Lanchester model accounts for the modeling of attrition in combat. This model has been extensively tested on empirical data, in particular using historical combat data, see Perry (2011) for a comprehensive study, and Lucas & Dinges (2004) for a specific application to the Battle of Kursk. Lanchester model's is a combat model, not a model for war, but Perry (2011) has empirically shown that the attrition effects at the battle level appeared to also take place at the more macro level of conflict.

and the spoils of war depend on the attacker's situation. These gains accrue to the attacker, but they are also subtracted from the defender's stock of capital. We expect that $\gamma > \rho$, so that the gains from predation are generally greater than the cost of waging war, although the relationship between these quantities may be dependent on the technological environment. In the case of unilateral war, the country under attack does not suffer defensive costs, since it is not waging war.

In bilateral war, however, both countries incur a pure defensive cost, without any gains, which is assumed to be a function of the attacker's power:

$$D(Y_{i,t}, Y_{-i,t}) = \tilde{\rho}_i Y_{-i,t} = \tilde{\rho}_i N_{-i} f_{-i}(k_{-i,t}),$$

with $\tilde{\rho} < 1$. This defensive cost can equivalently be seen as an increase in the cost of waging war, when the other country is also waging war at the same time.

The costs we have described above, when they are assessed against a country, are however subject to important limitations. The flow of defensive or offensive costs of war, which depend on the other country's capital stock, should not be able to go above some multiple m of the home country's total production. We hence consider that the intensity of predation towards Country i , or the intensity of war expenses, cannot exceed a rate of $m_i Y_{i,t} = m_i N_i f_i(k_{i,t})$. This effectively sets a limit to the speed at which a country can be pillaged, as a function of the size of its economy. Without such a limit, as a country's capital stock nears zero, expenses or predation, driven by the other country's capital, could reach below 0. Further, when a country incurs the costs of waging war at the same time as it suffers predation, we will assume that these costs are accounted for in priority: they are effectively subtracted from predation. Out of the maximum rate of pillage, predation can hence only apply to what is left after these expenses. Gains and costs are hence rewritten reflecting these various limits²:

- For the attacker in a unilateral war the cost of waging war per unit of time is capped as a function of the attacker's own production, and is equal to:

$$C(Y_{i,t}, Y_{-i,t}) \wedge m_i Y_{i,t} = \rho_i N_{-i} f_{-i}(k_{-i,t}) \wedge m_i N_i f_i(k_{i,t});$$

- In a unilateral war the effective gain of Country i attacking Country $-i$, per unit of time

²In order to reduce the length of expressions, we use the following standard mathematical short notations: $a \wedge b = \min(a, b)$, and $a \vee b = \max(a, b)$.

(equal to the loss of country $-i$ being attacked), is capped and computed as follows:

$$G(Y_{i,t}, Y_{-i,t}) \wedge m_{-i} Y_{i,t} = \gamma_i N_i f_i(k_{i,t}) \wedge m_{-i} N_{-i} f_{-i}(k_{-i,t});$$

- In a bilateral war, each country is an attacker relative to the other country, and incurs the cost of waging war C as well as the cost of defensive war D , which both need to be capped by the country's production. The cost of a bilateral war for Country i is therefore:

$$\begin{aligned} & (C(Y_{i,t}, Y_{-i,t}) + D(Y_{i,t}, Y_{-i,t})) \wedge m_i Y_{i,t} \\ & = (\rho_i + \tilde{\rho}_i) N_{-i} f_{-i}(k_{-i,t}) \wedge m_i N_i f_i(k_{i,t}); \end{aligned}$$

- From the standpoint of a country predated the other in a bilateral war, the total predation is capped as a function of the other country's production after it has been reduced by the costs above. As a result, from Country i 's standpoint, the gains from predation (also equal to the other country's loss to predation), are expressed as:

$$\begin{aligned} & G(Y_{i,t}, Y_{-i,t}) \wedge (m_{-i} Y_{-i,t} - (\rho_{-i} + \tilde{\rho}_{-i}) Y_{i,t}) \wedge m_{-i} Y_{-i,t} \\ & = \gamma_i Y_{i,t} \wedge [(m_{-i} Y_{-i,t} - (\rho_{-i} + \tilde{\rho}_{-i}) Y_{i,t}) \vee 0] \\ & = \gamma_i N_i f_i(k_{i,t}) \wedge [(m_{-i} N_{-i} f_{-i}(k_{-i,t}) - (\rho_{-i} + \tilde{\rho}_{-i}) N_i f_i(k_{i,t})) \vee 0]. \end{aligned}$$

If m_1 and m_2 are large enough, the full macrodynamics can be written in a clearer and relatively simple manner (the expressions in all generality depending on m_1 and m_2 are given in the mathematical appendix). In unilateral war, where Country 1 attacks Country 2:

$$\begin{cases} \frac{dk_{1,t}}{dt} = s_1 f_1(k_{1,t}) - \delta_1 k_{1,t} + \gamma_1 f_1(k_{1,t}) - \rho_1 \frac{N_2}{N_1} f_2(k_{2,t}) \\ \frac{dk_{2,t}}{dt} = s_2 f_2(k_{2,t}) - \delta_2 k_{2,t} - \gamma_1 \frac{N_1}{N_2} f_1(k_{1,t}) \end{cases}$$

In the case of bilateral war, the equations are symmetrical:

$$\begin{cases} \frac{dk_{1,t}}{dt} = s_1 f_1(k_{1,t}) - \delta_1 k_{1,t} - (\rho_1 + \tilde{\rho}_1) \frac{N_2}{N_1} f_2(k_{2,t}) + \gamma_1 f_1(k_{1,t}) - \gamma_2 \frac{N_2}{N_1} f_2(k_{2,t}) \\ \frac{dk_{2,t}}{dt} = s_2 f_2(k_{2,t}) - \delta_2 k_{2,t} - (\rho_2 + \tilde{\rho}_2) \frac{N_1}{N_2} f_1(k_{1,t}) + \gamma_2 f_2(k_{2,t}) - \gamma_1 \frac{N_1}{N_2} f_1(k_{1,t}) \end{cases}$$

Based on this model, we can identify steady-states depending on whether the countries are at war, in the case of two identical countries, but with potentially different population sizes.

Proposition 2.1 (Steady-State Equilibria). *Assuming two identical countries (apart from their population sizes and capital stock):*

- *If there is no war, the steady state capital k_n verifies $k_n = \frac{s}{\delta} f(k_n)$;*

- If there is unilateral war, the steady-state's expression depends on the parameters as follows:
 - If $\gamma + s < m$, the steady state capital levels for the attacker k_u and the defender k_d verify: $k_u = \frac{s+\gamma}{\delta} f(k_u) - \frac{\rho}{\delta} \frac{N_d}{N_u} f(k_d)$ and $k_d = \frac{s}{\delta} f(k_d) - \frac{\gamma}{\delta} \frac{N_u}{N_d} f(k_u)$. Capital levels verify $k_d < k_n$;
 - If $\gamma \wedge \frac{\rho\gamma}{s} > m$, then the steady-state levels are $k_u = \frac{s+\gamma-m}{\delta} f(k_u)$ and $k_d = \frac{s}{\delta} f(k_d) - \frac{\gamma}{\delta} \frac{N_u}{N_d} f(k_u)$. In addition, $k_d < k_n$ and $k_u > k_n$;
- If there is bilateral war, the symmetrical steady state capital k_b verifies $k_b = \frac{s-(\rho+\bar{\rho})\wedge m}{\delta} f(k_b)$. The bilateral war steady-state capital level verifies $k_b < k_n$.

Finally, if there is war (unilateral or bilateral), and if $f(0) = 0$, there are possible steady-states where one country is at k_n and the other is at zero.

Proof. The proof is given in the mathematical appendix. □

As an illustration, if we pick $f(x) = \sqrt{x}$, then we can easily solve some of these expressions analytically, and we have:

$$k_n = \frac{s^2}{\delta^2}$$

$$k_b = \frac{(s - (\rho + \bar{\rho}) \wedge m)^2}{\delta^2},$$

and if m is small enough, and $\gamma \wedge \frac{\rho\gamma}{s} > m$, we also have:

$$k_u = \frac{(s + \gamma - m)^2}{\delta^2}$$

$$k_d = \frac{1}{4\delta^2} \left(s + \sqrt{s^2 - 4 \frac{N_u}{N_d} \gamma (s + \gamma - m)} \right)^2.$$

In the case of bilateral war, while the steady-state capital per person may be the same, total production will differ between the two countries if their population sizes differ. We can observe the effect of the ratio of population sizes on the steady-states, in the case of unilateral war. As N_u increases relative to N_d , then k_u generally increases, and k_d declines.

With reasonable parameters, for two identical countries, not being an attacker in war, or being engaged in a bilateral war, are both worse than being at peace from an economics perspective. At this juncture, we will assume that the parameters are such that there is a non-zero k_n that solves $k_n = \frac{s}{\delta} f(k_n)$, and $k_u - k_n > 0$, so that there is some fundamental benefit in attacking the other country, otherwise then there could not be any war.

Proposition 2.2 (Number and Existence of Steady-States). *We assume that both countries*

have the same population, so that $N_1 = N_2$, that m is large, and that $f'(0) > \frac{\delta}{s-\rho-\bar{\rho}}$.

- Steady-state capital levels k_b and k_n , for bilateral war or no war, are unique;
- In the case of unilateral war, there may be 0, 1 or 2 non-trivial steady-state equilibria depending on the parameters and on the shape of the production function f , and on the value of $\frac{s}{\rho}(s + \gamma) - \gamma$.

Proof. The proof is given in the mathematical appendix. □

In some particular conditions, the steady-states are locally stable. We have:

Proposition 2.3 (Local Stability of Steady-States). *We assume that both countries have the same population, so that $N_1 = N_2$, and that m is large. Around the steady-state:*

- In the case of no war, the system is always locally stable;
- In the case of bilateral war, if $(s + 2\gamma + \rho + \bar{\rho})f'(k) < \delta$ then the system is locally stable;
- In the case of unilateral war, if

$$(s + \gamma) f'(k_u) + s f'(k_d) < \delta + \left(\delta \wedge \frac{(s + \gamma) s - \rho (\gamma + \bar{\rho})}{\delta} f'(k_u) f'(k_d) \right)$$

then the system is locally stable.

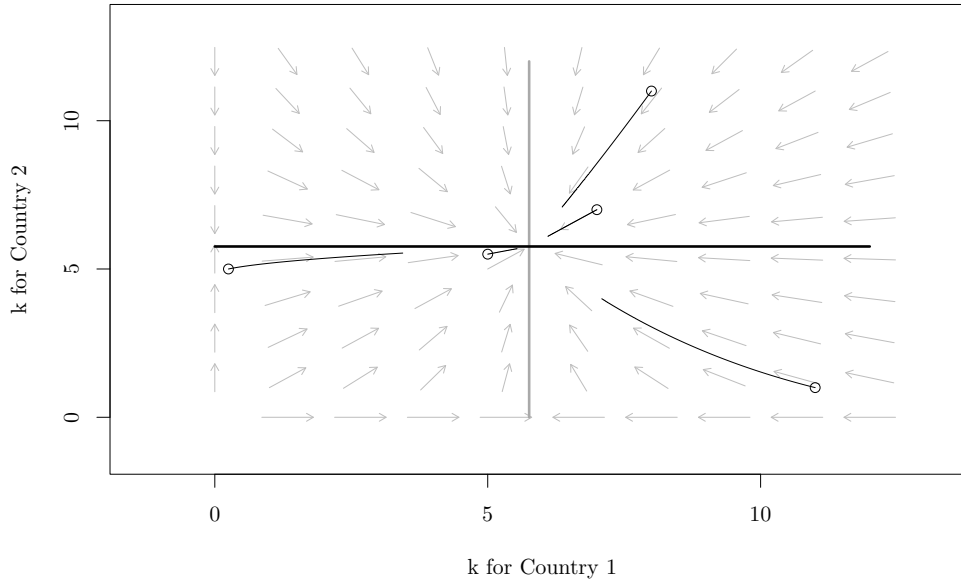
Proof. The proof is given in the mathematical appendix. □

The steady-state dynamics are useful as they inform us on how military decisions will translate into long term economic evolutions, but we do not know yet how they may be reached by the system. In order to explore the dynamics for these different configurations between the two countries, we now turn to phase diagram analysis. In order to simplify the numerical analysis, we will further assume that $N_1 = N_2$ in these examples.

We can easily plot the phase diagrams when there is no war, with bilateral war, or with unilateral war, assuming for now that both countries stay in the same configuration, and do not stop, or start a war. Following Grayling (2014), we use the R package phaseR for this purpose (Burger, Grayling, & Capretto 2022). Figure 1 shows a numerical example for the no-war simple case. The plot is expressed in the space (k_1, k_2) of all combinations of capital for Country 1 and Country 2. The grey and black thick lines show the nullclines for Country 1 and Country 2, respectively, that is the curves where the differentials $\frac{dk_1}{dt}$ or $\frac{dk_2}{dt}$ are zero. The flow field, represented by arrows, shows the vectors of these differentials $\left(\frac{dk_1}{dt}, \frac{dk_2}{dt} \right)$ across the plane. Finally, the thin lines are example trajectories across time, starting from the points in the small

circles. When there is no war, there is a clear convergence towards to long term equilibrium where the nullclines intersect, whatever the starting conditions, solving $k_n = \frac{s}{\delta} f(k_n)$, which, with our choice of f in the chart, gives $k_n = \frac{s^2}{\delta^2} = 5.76$.

Figure 1: *No War, or Perpetual Peace*



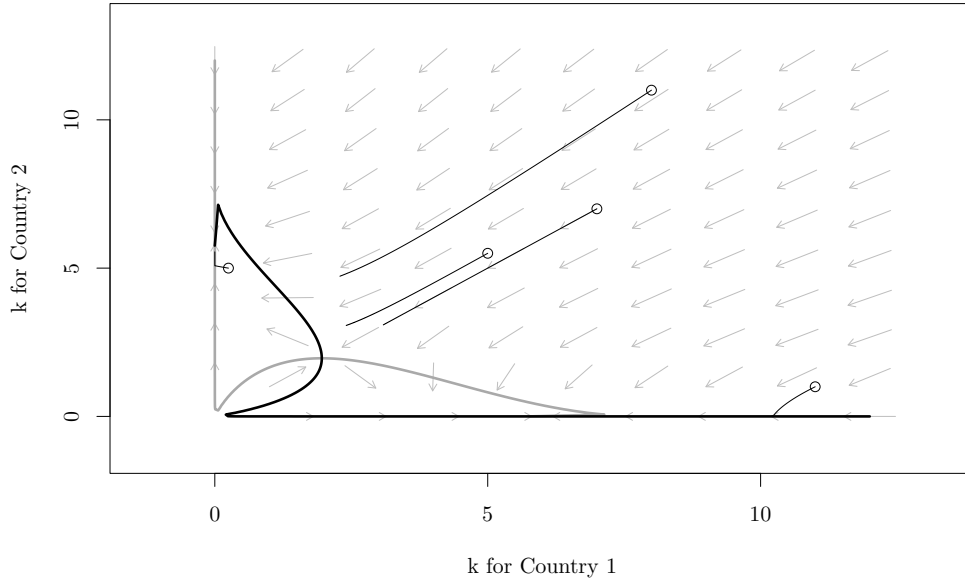
Note: The parameters are defined as follows: $m = 1$, $s = 0.12$, $\gamma = 0.02$, $\delta = 0.05$, $\rho = 0.03$, $\bar{\rho} = 0.02$. The production function is $f(x) = \sqrt{x}$. Small circles and thin lines represent trajectories; the thicker grey line is Country 1's nullcline, and the thicker black line is Country 2's nullcline.

As an illustration of Proposition 2.2, Figure 2 shows us that when there is sustained bilateral war, then the steady state where both countries reach the same capital $k_b < k_n$ can only be attained if both countries started from the exactly the same situation. As the example trajectories show, the country with the greater starting capital will end up losing some capital, while the other country will end up at zero. We can see that $(0, 0)$ is also a steady state although it does not seem stable. With our choice of f in the chart, we have $k_b = \frac{(s - (\rho + \bar{\rho}) \wedge m)^2}{\delta^2} = 1.96$

In the case of unilateral war, Figure 3 shows us that, when predation γ is relatively low, there are three possible steady states, apart from $(0, 0)$, according to the conditions from Proposition 2.2. The main equilibrium, to which most trajectories appear to converge, leads to a capital level k_u slightly greater than the no-war case for the attacker, while for the defender k_d is lower than k_n . The other two steady states are high capital for the attacker and very low capital for the defender, or reciprocally. The latter two states do not appear to be stable however. These three steady states appear because of the way in which the nullclines's shapes intersect.

If the gains from predations are higher, as illustrated in Figure 4 where $\gamma = 0.05$, then the nullcline curve for the attacker expands massively, while it contracts for the defender, which

Figure 2: *Perpetual Bilateral War*



Note: The parameters are defined as follows: $m = 1$, $s = 0.12$, $\gamma = 0.02$, $\delta = 0.05$, $\rho = 0.03$, $\bar{\rho} = 0.02$. The production function is $f(x) = \sqrt{x}$. Small circles and thin lines represent trajectories; the thicker grey line is Country 1's nullcline, and the thicker black line is Country 2's nullcline.

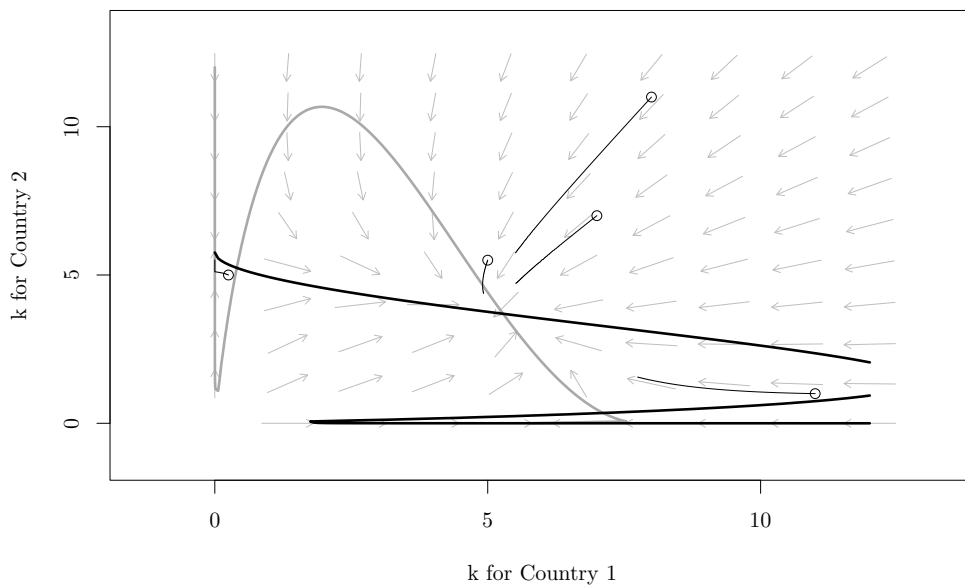
results in only two steady states: $(0, 0)$, and a very low capital level for the attacker, and close to k_n for the defender. This state is however not stable, and we can see that most trajectories converge to zero capital for the defender, and a high level of capital for the attacker. As predation increases, the black nullcline is compressed to the left and the grey nullcline is magnified, so that the middle steady-state shifts towards the right (higher k_u) and lower (lower k_d), until the curves do not intersect anymore, apart from the low k_1 steady-state.

This core macrodynamic model has allowed us to capture the combined effects of economic output and war, leading to a variety of long-run equilibria, some of which involve a country's capitulation, depending on the relative sizes, capital stock, and economic or military efficiency of these countries. In relation to the model from Lagerlöf (2010), not accounting for the details of the economic impact of land grabbing or human loss has allowed us to qualify the steady-state equilibria in all generality.

2.2 Value of Life, Political Regime, and War Decision

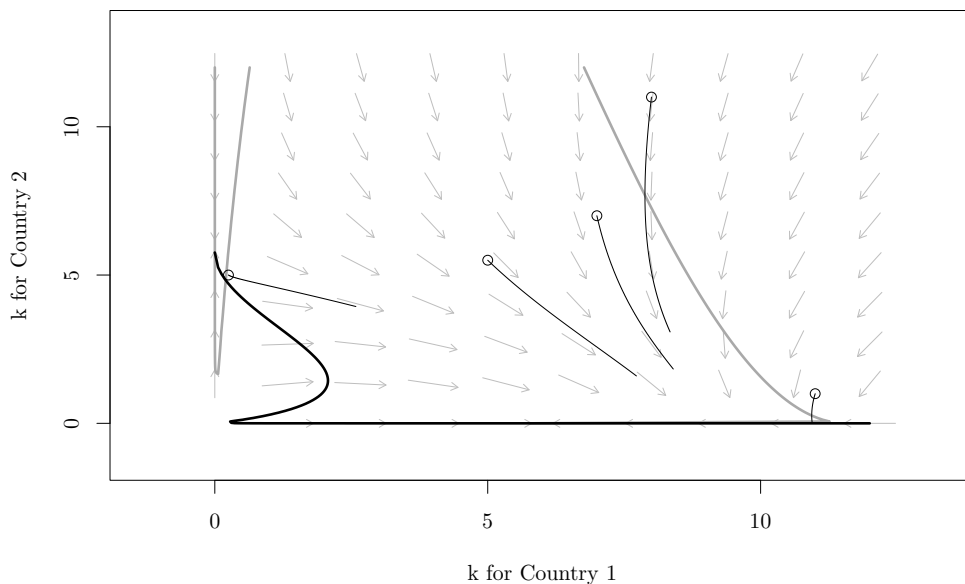
While the discussion above addressed pure economic dynamics, we now turn to the elements that are factored into decisions. The decision to go to war or not, or to stop an on-going war or not, should depend on the economic trade-off between being at war and not being at war. However, we argue that, in all generality, the decision cannot be based purely on

Figure 3: *Unilateral War (Country 1 Attacks)*



Note: The parameters are defined as follows: $m = 1$, $s = 0.12$, $\gamma = 0.02$, $\delta = 0.05$, $\rho = 0.03$, $\bar{\rho} = 0.02$. The production function is $f(x) = \sqrt{x}$. Small circles and thin lines represent trajectories; the thicker grey line is Country 1's nullcline, and the thicker black line is Country 2's nullcline.

Figure 4: *Unilateral War with High Predation Gains (Country 1 Attacks)*



Note: The parameters are defined as follows: $m = 1$, $s = 0.12$, $\gamma = 0.05$, $\delta = 0.05$, $\rho = 0.03$, $\bar{\rho} = 0.02$. The production function is $f(x) = \sqrt{x}$. Small circles and thin lines represent trajectories; the thicker grey line is Country 1's nullcline, and the thicker black line is Country 2's nullcline.

economic grounds, and also factors in the human cost of war, $H_i(L) = v_{i,d}L_i + v_{i,f}L_{-i}$. The parameters $v_{i,d}$ and $v_{i,f}$ express, respectively, the cost of domestic human life and the cost of foreign human life for country i . Although Eckhardt (1991) found an increasing trend in the proportion of war deaths over time, according to the broader and more recent study by Oka et al. (2017), the scale of war engagements across all history, and the scale of war deaths, corresponding to L in this model, have generally evolved in line with overall population size³. In the context of the model, the difference in populations between war and peace L may also not strictly represent deaths in combat or caused by war, but also the part of the population that is pulled away from productive contribution to the economy, for example through mobilization.

From a pure economic standpoint, the difference in $\frac{dk_{i,t}}{dt}$ between being at peace and attacking the other country $-i$ unilaterally, if the other country does not fight, is equal to:

$$\gamma_i f_i(k_{i,t}) \wedge m_{-i} \frac{N_{-i}}{N_i} f_{-i}(k_{-i,t}) - \rho_i \frac{N_{-i}}{N_i} f_{-i}(k_{-i,t}) \wedge m_i f_i(k_{i,t}).$$

If the other country fights, then the difference is equal to:

$$\begin{aligned} & \gamma_i f_i(k_{i,t}) \wedge \left[\left(m_{-i} \frac{N_{-i}}{N_i} f_{-i}(k_{-i,t}) - (\rho_{-i} + \tilde{\rho}_{-i}) f_i(k_{i,t}) \right) \vee 0 \right] \\ & - \gamma_{-i} \frac{N_{-i}}{N_i} f_{-i}(k_{-i,t}) \wedge \left[\left(m_i f_i(k_{i,t}) - (\rho_i + \tilde{\rho}_i) \frac{N_{-i}}{N_i} f_{-i}(k_{-i,t}) \right) \vee 0 \right] \\ & + \gamma_{-i} \frac{N_{-i}}{N_i} f_{-i}(k_{-i,t}) \wedge m_i f_i(k_{i,t}) \\ & - (\rho_i + \tilde{\rho}_i) \frac{N_{-i}}{N_i} f_{-i}(k_{-i,t}) \wedge m_i f_i(k_{i,t}). \end{aligned}$$

Reflecting the human cost of war H , we can define the trade-off of breaking the peace by starting unilateral war as:

$$\begin{aligned} TO_i^u(k_t, L) &= \gamma_i f_i(k_{i,t}) \wedge m_{-i} \frac{N_{-i}}{N_i} f_{-i}(k_{-i,t}) \\ & - \rho_i \frac{N_{-i}}{N_i} f_{-i}(k_{-i,t}) \wedge m_i f_i(k_{i,t}) - v_{i,d}L_i - v_{i,f}L_{-i}, \end{aligned}$$

³Keenan-Jones & Hebblewhite (2019) raised a substantial issue with Oka et al. (2017) concerning Antiquity, pointing out the dangers in believing the data from ancient historians without any critique. Nevertheless, for the modern and contemporary periods, for which many more data points are available, the results from Oka et al. (2017) appear valid.

and the trade-off related to a defensive position as:

$$\begin{aligned}
TO_i^d(k_t, L) = & \gamma_i f_i(k_{i,t}) \wedge \left[\left(m_{-i} \frac{N_{-i}}{N_i} f_{-i}(k_{-i,t}) - (\rho_{-i} + \tilde{\rho}_{-i}) f_i(k_{i,t}) \right) \vee 0 \right] \\
& - \gamma_{-i} \frac{N_{-i}}{N_i} f_{-i}(k_{-i,t}) \wedge \left[\left(m_i f_i(k_{i,t}) - (\rho_i + \tilde{\rho}_i) \frac{N_{-i}}{N_i} f_{-i}(k_{-i,t}) \right) \vee 0 \right] \\
& + \gamma_{-i} \frac{N_{-i}}{N_i} f_{-i}(k_{-i,t}) \wedge m_i f_i(k_{i,t}) \\
& - (\rho_i + \tilde{\rho}_i) \frac{N_{-i}}{N_i} f_{-i}(k_{-i,t}) \wedge m_i f_i(k_{i,t}) - v_{i,d} L_i - v_{i,f} L_{-i}.
\end{aligned}$$

We consider that a country will go to war if $TO_i^u(k_t, L) > 0$ and $TO_i^d(k_t, L) > 0$, as an instantaneous decision: one attacks if one is sure to gain from attacking whatever the other country decides to do.

Note that if m_i and m_{-i} are large, then the expressions reduce to:

$$TO_i^u(k_t, L) = \gamma_i f_i(k_{i,t}) - \rho_i \frac{N_{-i}}{N_i} f_{-i}(k_{-i,t}) - v_{i,d} L_i - v_{i,f} L_{-i},$$

and:

$$TO_i^d(k_t, L) = \gamma_i f_i(k_{i,t}) - (\rho_i + \tilde{\rho}_i) \frac{N_{-i}}{N_i} f_{-i}(k_{-i,t}) - v_{i,d} L_i - v_{i,f} L_{-i}.$$

In this case, the conditions $TO_i^u(k_t, L) > 0$ and $TO_i^d(k_t, L) > 0$ reduce to $TO_i^d(k_t, L) > 0$.

This parameterization with the function $H_i(L)$ is in fact directly related to the political regime for each country: autocracies do not value life, while democracies would be expected to. The game-theoretical model of autocracy as opposed to democracy proposed by Besley & Kudamatsu (2008) shows that autocrats only cater to a small group of people who help maintain them in power, while in a democratic system the leaders must attend to the needs of the whole population. In consequence, it is logical that the value of population life in autocracies should be lower than in democracies. Further, Gartner (2008) shows, based on surveys and experimental tests carried out in the US on the Iraq war, that the number of casualties and the trend in casualties are strong drivers of the support the population gives to war. In a democratic society, greater or diminished support of the population directly translates into political decision, while this may not be the case in a non-democratic society.

Müller (2004) has shown that modern democracies may in fact enter wars in order to protect other foreign populations, and proposed a distinction between “militant” and “pacifist” democracies. For Appadurai (2007), the meaning of equality as a fundamental part of democracy has been enhanced, to be projected onto populations who did not have it. In the case of the “militant” democracies, the parameters v_d and v_f are therefore both positive, and

may have the same magnitude. As N. P. Gleditsch & Hegre (1997) stressed, “There is a great deal of other evidence that democracies value human life more highly, notably because of the absence in democracies of genocide [...] and famine [...] and lower incidence of civil war [...]” (p. 294). Stressing the importance of democracy as a universal value, Sen (1999) points out that “The value of democracy includes its intrinsic importance in human life” (p. 16), because it offers each citizen the occasion to participate and affect the regime. Carter (2017) has also empirically shown that democracies were more reluctant than autocracies to mobilize, in particular because of the political cost for their leaders. Further, Goenner (2007) showed empirically that modern democracies prefer to apply economic sanctions, rather than go to war, as a substitute that allows them to avoid human loss.

We will hence categorize regimes in three buckets:

- The autocracy, for which human life has no value and $v_d = v_f = 0$, denoted by a subscript A for the country index⁴;
- The nationalist democracy, for which only domestic human life matters, and $v_d > 0$ but $v_f = 0$, denoted by a subscript D for the country index;
- The universalist democracy, for which all life matters and v_d and $v_f > 0$, with possibly $v_d = v_f$, denoted by a subscript U for the country index.

Proposition 2.4 (Regime and Instantaneous War Decision). *Assume identical countries, with $L = L_i = L_{-i}$, and f strictly increasing. Also assume that $\gamma N_i f(k_i) < m N_{-i} f(k_{-i})$, and $(\rho + \bar{\rho}) N_{-i} f(k_{-i}) < m N_i f(k_i)$ are verified. Then, the conditions leading Country i to enter unilateral war at time t , depending on the regime category, can be expressed as a threshold relationship between the capital of both countries, of the form $k_{-i,t} < e_i(k_{i,t})$, where:*

- For an autocracy, $e_A(x) = f^{-1}\left(\frac{\gamma}{(\rho + \bar{\rho})} \frac{N_i}{N_{-i}} f(x)\right)$;
- For a nationalist democracy, $e_D(x) = f^{-1}\left(\frac{\gamma}{(\rho + \bar{\rho})} \frac{N_i}{N_{-i}} f(x) - \frac{v_{D,d}}{(\rho + \bar{\rho})} \frac{N_i}{N_{-i}} L\right)$;
- For a universalist democracy, $e_U(x) = f^{-1}\left(\frac{\gamma}{(\rho + \bar{\rho})} \frac{N_i}{N_{-i}} f(x) - \frac{v_{U,d} + v_{U,f}}{(\rho + \bar{\rho})} \frac{N_i}{N_{-i}} L\right)$.

Further, we have $e_U(x) < e_D(x) < e_A(x)$.

Proof. Given our assumption, both instantaneous trade-off conditions for going to war reduce to $TO_i^d(k_t, L) > 0$, with:

$$TO_i^d(k_t, L) = \gamma f(k_{i,t}) - (\rho + \bar{\rho}) \frac{N_{-i}}{N_i} f(k_{-i,t}).$$

⁴Even if we use this expression, we will still write $-i$ for the other country.

Hence $TO_A^d(k_{A,t}, L) > 0$, for the decision leading to war for an autocracy, writes:

$$\gamma f(k_{A,t}) - (\rho + \tilde{\rho}) \frac{N_{-i}}{N_i} f(k_{-i,t}) > 0.$$

This is equivalent to $k_{-i,t} < f^{-1}\left(\frac{\gamma}{(\rho + \tilde{\rho})} \frac{N_i}{N_{-i}} f(k_{A,t})\right)$, that is $k_{-i,t} < e_A(k_{A,t})$ with $e_A(x) = f^{-1}\left(\frac{\gamma}{(\rho + \tilde{\rho})} \frac{N_i}{N_{-i}} f(x)\right)$, where e designates the entry to war threshold. The greater the population of the country that one intends to attack, the lower the capital stock per person needs to be.

For a nationalist democracy D , where $v_d > 0$, we have the condition for unilateral war $TO_D^d(k_{D,t}, L) > 0$, which reduces to

$$\gamma f(k_{D,t}) - (\rho + \tilde{\rho}) \frac{N_{-i}}{N_i} f(k_{-i,t}) - v_{D,d} L > 0.$$

This is equivalent to: $k_{-i,t} < e_D(k_{D,t})$ with

$$e_D(x) = f^{-1}\left(\frac{\gamma}{(\rho + \tilde{\rho})} \frac{N_i}{N_{-i}} f(x) - \frac{v_{D,d}}{(\rho + \tilde{\rho})} \frac{N_i}{N_{-i}} L\right).$$

For a universalist democracy U , where $v_d > 0$ and $v_f > 0$, following the same logic, we obtain that the decision to enter a unilateral war is $k_{-i,t} < e_U(k_{U,t})$ with

$$e_U(x) = f^{-1}\left(\frac{\gamma}{(\rho + \tilde{\rho})} \frac{N_i}{N_{-i}} f(x) - \frac{v_{U,d} + v_{U,f}}{(\rho + \tilde{\rho})} \frac{N_i}{N_{-i}} L\right).$$

From these expression, and since f is strictly increasing, we can immediately see that:

$$e_U(x) < e_D(x) < e_A(x),$$

and this concludes the proof. □

We see that an autocracy is hence more likely to enter war, followed by a nationalist democracy and a universalist democracy. If the other country's capital per person falls under some level, the attacker is expected to instantly wage war.

The macrodynamics we examined earlier should then be affected, as a function of these trade-off considerations. Given a combination of capital levels (k_1, k_2) at a point in time, each country can determine, based on its specific human life cost, whether it is best to wage war or not. The equations driving the phase diagrams we displayed previously can therefore be modified, so that they reflect the war/peace decision.

We will assume that each country considers its trade-off along the lines of Proposition 2.4,

if m is large, in an instantaneous manner: a country goes to war, or stays in war if the gain from waging war over not waging war is greater, whether the other country attacks or not. Considering identical countries, apart from the political regime, each country i therefore wages war if:

$$\gamma f(k_{i,t}) - (\rho + \tilde{\rho}) \frac{N_{-i}}{N_i} f(k_{-i,t}) - v_i L > 0,$$

where v_i is the weight allocated to human loss L (the same loss for both countries). The fact that war may take place depends on the weights v_1 and v_2 each country associates to human loss in war.

Proposition 2.5 (Decision and Steady-State Consistency). *We assume that m is large, and $N_1 = N_2$.*

- If $k_n < (v_1 \wedge v_2) \frac{sL}{\delta(\gamma - \rho - \tilde{\rho})}$, then there is a steady-state with no war;
- If $k_b > (v_1 \vee v_2) \frac{(s - \rho - \tilde{\rho})L}{\delta(\gamma - \rho - \tilde{\rho})}$, then there is a steady-state with bilateral war;
- If

$$\begin{cases} (s - \rho - \tilde{\rho})f(k_d) - \delta k_d > v_u L \\ (\gamma(s + \gamma) - \rho(\rho + \tilde{\rho}))f(k_u) - \delta \gamma k_u < \rho v_d L \end{cases}$$

then there is a steady-state where Country u attacks Country d .

Proof. We consider the steady-state equations for the various situations we are interested in (no war, bilateral war, unilateral war), and see what conditions are implied on the parameters for these steady-states to be consistent with the decision rule.

If there is no war, then we know that $k_1 = k_2 = k_n$. The situation is compatible with the decision logic if $\gamma f(k_i) - (\rho + \tilde{\rho})f(k_{-i}) < v_i L$ for both countries. Hence we must have: $\gamma f(k_n) - (\rho + \tilde{\rho})f(k_n) < v_1 L$ and $\gamma f(k_n) - (\rho + \tilde{\rho})f(k_n) < v_2 L$. However, we know that by definition $k_n = \frac{s}{\delta} f(k_n)$, so that both conditions are equivalent to:

$$k_n < (v_1 \wedge v_2) \frac{sL}{\delta(\gamma - \rho - \tilde{\rho})}.$$

If there is bilateral war, then we have at the steady-state $k_1 = k_2 = k_b$. The decision conditions for bilateral war to take place are that $\gamma f(k_i) - (\rho + \tilde{\rho})f(k_{-i}) > v_i L$ for both countries. Hence, we must have $\gamma f(k_b) - (\rho + \tilde{\rho})f(k_b) > v_1 L$ and $\gamma f(k_b) - (\rho + \tilde{\rho})f(k_b) > v_2 L$. However we know that $k_b = \frac{s - \rho - \tilde{\rho}}{\delta} f(k_b)$, so that both conditions sum up with:

$$k_b > (v_1 \vee v_2) \frac{(s - \rho - \tilde{\rho})L}{\delta(\gamma - \rho - \tilde{\rho})}.$$

In the case of unilateral war, we have at the steady-state $k_u = \frac{s+\gamma}{\delta} f(k_u) - \frac{\rho}{\delta} f(k_d)$ and $k_d = \frac{s}{\delta} f(k_d) - \frac{\gamma}{\delta} f(k_u)$. The conditions for the unilateral war to be consistent with each country's decisions are that $\gamma f(k_u) - (\rho + \tilde{\rho}) f(k_d) > v_u L$ and $\gamma f(k_d) - (\rho + \tilde{\rho}) f(k_u) < v_d L$. Applying the equations for k_u and k_d and substituting, we obtain the following conditions:

$$\begin{cases} (s - \rho - \tilde{\rho}) f(k_d) - \delta k_d > v_u L \\ (\gamma(s + \gamma) - \rho(\rho + \tilde{\rho})) f(k_u) - \delta \gamma k_u < \rho v_d L \end{cases}$$

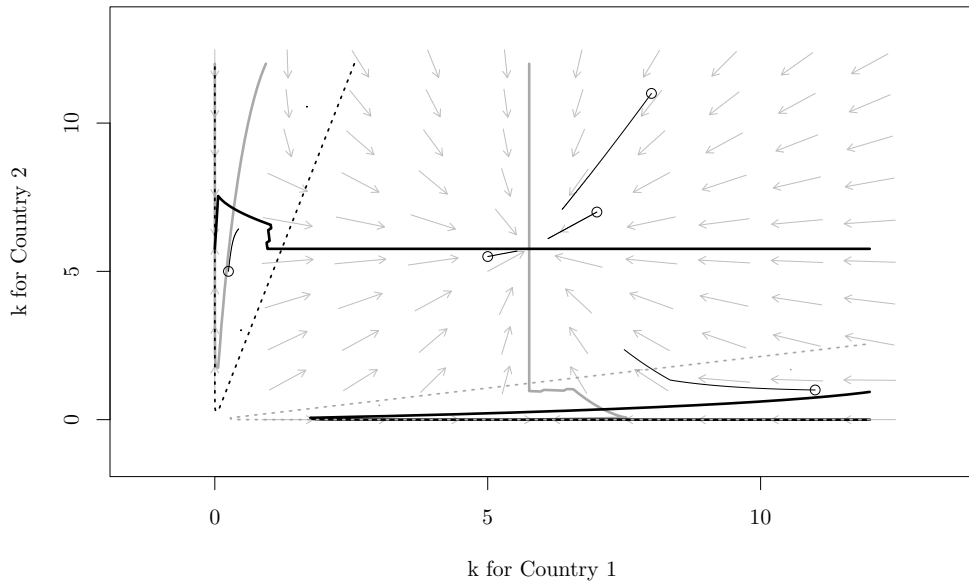
If the attacker's aversion to human loss is low and the defender's aversion to human loss is high, we can see that the unilateral war steady-state is more likely to exist. \square

We examine a few numerical examples, where we will assume $N_1 = N_2$. Figure 5 displays the phase diagram for the case of two autocracies ($v_1 = v_2 = 0$), with the same parameters as before. In addition to the previous diagrams, we also display with dotted lines (grey for Country 1, black for Country 2) the decision threshold between war and peace: South-East of the grey dotted line means that Country 1 will wage war; North-West of the black dotted line, Country 2 will enter war. In the case at hand, where the gains from predation are relatively modest as in prior examples ($\gamma = 0.02$), in the whole central area there is no war, and as a result an equilibrium steady-state is possible. Several trajectories converge to that point, without crossing the peace/war frontier. We can also see other possible steady-state conditions, with some gains from one country above the peace equilibrium, and deep losses for the other one. Given the shape of the flow field, these other steady states seem to have small attraction basins, and are hence probably not very realistic. In this case, k_n is large enough that the conditions of the first part of Proposition 2.5 are verified, and there is a no-war steady state consistent with both countries' decisions.

If we change γ to a higher value, one for which, incidentally and thanks to Figure 4, we know that there is no stable unilateral war equilibrium, Figure 6 shows us the overall outcome. While a central area is still defined by the war/peace decision thresholds, they are now inverted: Country 1 wages war everywhere South-East of the dotted grey line, and Country 2 wages war North-West of the black dotted line: the intersect of these areas, the whole center area, corresponds to bilateral war. We can see that in this area, the trajectories resemble the ones we observed in Figure 2. In the areas beyond the thresholds, only one country is at war and the other one gets driven to a capital of zero. In this case, the second part of Proposition 2.5 is verified, because γ is high.

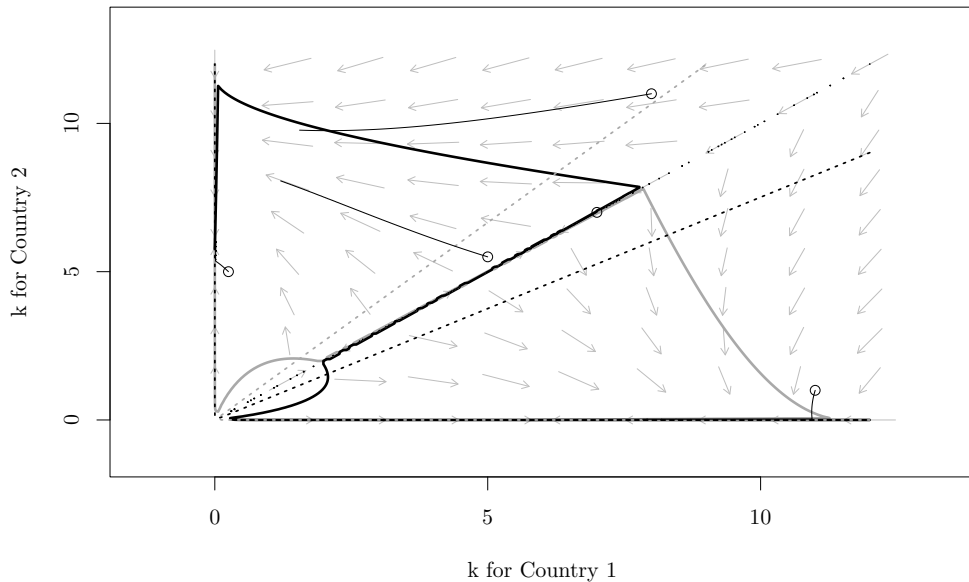
Still in the case of high predation gains ($\gamma = 0.05$), as in the prior example, we now compare a democracy ($v_1 > 0$) with an autocracy ($v_2 = 0$). Figure 7 shows the phase diagram, and

Figure 5: *Dynamic War Decision Between Autocracies: Stable Perpetual Bilateral War*



Note: The parameters are defined as follows: $m = 1$, $s = 0.12$, $\gamma = 0.02$, $\delta = 0.05$, $\rho = 0.03$, $\bar{\rho} = 0.02$, $L = 0.1$, $v_1 = v_2 = 0$. The production function is $f(x) = \sqrt{x}$. Small circles, thin lines: trajectories; dotted: decision threshold, thick: nullclines; grey: Country 1, black: Country 2.

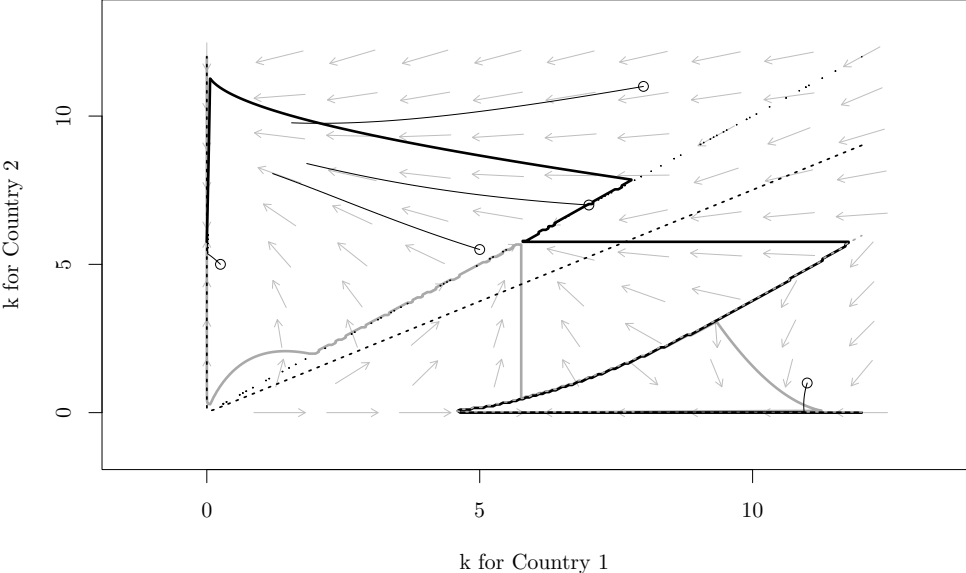
Figure 6: *Dynamic War Decision Between Autocracies with High Predation Gains: Unstable Perpetual Peace*



Note: The parameters are defined as follows: $m = 1$, $s = 0.12$, $\gamma = 0.05$, $\delta = 0.05$, $\rho = 0.03$, $\bar{\rho} = 0.02$, $L = 0.1$, $v_1 = v_2 = 0$. The production function is $f(x) = \sqrt{x}$. Small circles, thin lines: trajectories; dotted: decision threshold, thick: nullclines; grey: Country 1, black: Country 2.

we can observe it is very different from the case of conflict between two autocracies. The autocracy, here Country 2, does not change its war/peace threshold, the black dotted line, from what we observed in Figure 6, but Country 1, now a democracy, is much more reluctant to go to war because of its aversion to human loss. As a result, the decision threshold for Country 1, the grey dotted line, is now much lower and to the right. Hence there now exists an area, under the grey dotted line and above the black dotted line, where there is no war, in which some trajectories may converge to the steady-state point. For most of the area in this domain, however, the democracy does not wage war, and the autocracy does, which results in unilateral war, bringing the democracy's capital down to zero.

Figure 7: *Dynamic War Decision Between a Democracy and an Autocracy with High Predation Gains: Fragility of Perpetual Peace*

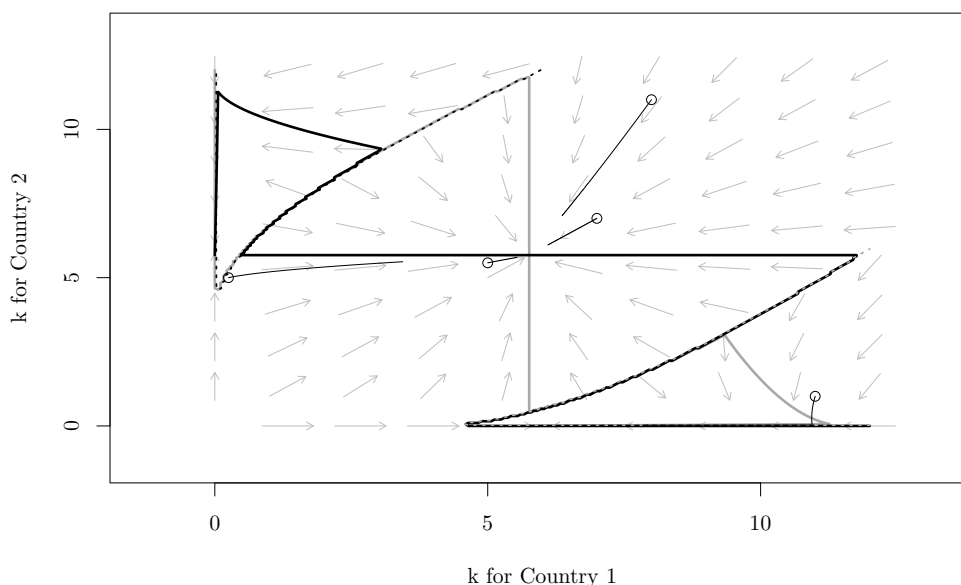


Note: The parameters are defined as follows: $m = 1$, $s = 0.12$, $\gamma = 0.05$, $\delta = 0.05$, $\rho = 0.03$, $\bar{\rho} = 0.02$, $L = 0.1$, $v_1 = 1.0$, $v_2 = 0$. The production function is $f(x) = \sqrt{x}$. Small circles, thin lines: trajectories; dotted: decision threshold, thick: nullclines; grey: Country 1, black: Country 2.

Finally, we can consider the case of two democracies facing each other, as shown in Figure 8, where $v_1 = v_2 > 0$. The democracies's choices lead them to avoid war, because of the human cost, unless waging war becomes very attractive, at the sides of the threshold curves. Therefore, the center area corresponds to a peace equilibrium, and we can see that it is stable across a large domain. When there is unilateral war, depending on k_1 and k_2 , the country under attack loses all its capital while the other country may gain slightly. In spite of the fact that γ is large, the higher parameter v allows the first part of Proposition 2.5 to be verified, and there is a no-war steady-state.

By extending the model with the instantaneous decision to enter war or not, and the perceived

Figure 8: *Dynamic War Decision Between Democracies with High Predation Gains: Favourable Conditions for Stable Perpetual Peace*



Note: The parameters are defined as follows: $m = 1$, $s = 0.12$, $\gamma = 0.05$, $\delta = 0.05$, $\rho = 0.03$, $\bar{\rho} = 0.02$, $L = 0.1$, $v_1 = v_2 = 1.0$. The production function is $f(x) = \sqrt{x}$. Small circles, thin lines: trajectories; dotted: decision threshold, thick: nullclines; grey: Country 1, black: Country 2.

cost of human loss, we have effectively offered a formal proof of Kant’s “theorem”: the more democratic the country, the least likely it is to enter war. Even as the potential gains from predation increase, to such a point that there would be continuous war between autocracies, democracies will refrain from entering war in the model because of their greater aversion to human loss. This additional degree of freedom in the model therefore allows us to account for a larger range of stylized facts than the models from Lagerlöf (2010) and Hess & Orphanides (2001).

3 Applications of the Model to History

In this section, we discuss how the model’s parameters may account for a large variety of wars and conflict situations. We address particular features of the model separately, in order to show how they can account for these situations. As we have seen in the literature review in Section 1, very many empirical models have looked into, and qualified, Kant’s theory of war and democracy. Hence, we do not seek to offer another empirical validation, but rather to show how our model can account for this pattern within various situations, along the lines of the discussions in Alesina, Reich, & Riboni (2020) or in Acemoglu et al. (2012). Indeed, the relevance of our model can be best captured by the fact that it allows us to

account for the specificity of a wide range of historical conflicts. We envision the model as a Weberian *Idealtypus*, a compact way of thinking through war. We begin with a discussion of the measures that capture the gains in predation and the costs in waging war, including the impact of technology and weapon improvements. Then, we address the cost of human life as a function of the political regime.

3.1 Opportunity and Cost of War

In addition to differences in country size, the parameters that drive a country's ability to reap gains from another, or cause their costs to rise, are significant drivers of the propensity to wage war in our model. We explore the meaning of these parameters in greater detail, relating them to particular historical situations.

The parameters that drive the efficiency with which one country may benefit from attacking another, γ , ρ and $\tilde{\rho}$, are largely related to the available technology. Some technological improvements may be easily transferable and hence only provide a short-lived advantage to a country, while others require a substantial infrastructure to be properly used. For example, about the Spaniards in South America, "Their cannons, arquebuses and crossbows all had a greater range and killing power than their rivals' weapons. In addition, the Aztecs probably could not use captured firearms, because they had no gunpowder and also lacked the necessary training" (Black [2002] 2005, 61). Similarly, the development of gunpowder-based siege cannons in the early 14th century in Europe was not something that could be easily copied without an important investment in skills and material resources. The development of stirrups in the early Middle Ages, on the other hand, was easily transferred. However, one of the important military consequences of the use of stirrups was the ability to later organize destructive cavalry charges by heavily armored knights, and the skills and resources necessary to build such armors were not so easily transferred (DeVries 1992). The Northern Crusades in the 13th century are a good example of the military efficiency of Teutonic knights against lightly armored fighting troops (Urban 1998). In ancient and medieval history, rapid changes in γ or ρ were therefore not likely within a generation, but could take place over a few generations, allowing for a progressive balancing of the scales. Differences in population growth taking place over the course of a generation, due to differences in local agricultural productivity or to localized famines, for example, could hence be potential drivers of war incentives.

The gains and costs associated with predation in unilateral war can represent the case where one country exploits another one, while the other country is not putting up a military defense. For example, when Sparta effectively enslaved Messenia in the 8th century BCE, that region's

production was largely preempted by the Spartans, allowing them to fund the martial training of their citizens, who did not have to work the land themselves (Cartledge 2002). This situation may be represented as unilateral war within the model, eventually turning into bilateral war when the Messenians revolted at the end of the 7th century BCE. Another example in which the unilateral war dynamics may be used to represent a situation without an actual military conflict is that of a regular tribute : in the 9th century CE, the Carolingians expected to obtain significant tribute from the Saxons or the Thuringians, and while they did there was no war—but this would be represented as unilateral war in the model; if they did not pay then there was bilateral war (Reuter 1985). More recently, the 1919 Treaty of Versailles and the subsequent 1921 London Schedule of Payments required Germany to pay billions of dollars in war damages, and France occupied the Ruhr region in order to enforce these payments. This situation, from our model’s perspective, could be associated to a defensive war. In these situations, if the exploited country’s ability to wage war (through γ) increased, then it could become optimal to enter war. In the case of the Messenians, Sparta enlisted many during various foreign wars, which provided them with an access to arms and training. Germany, in spite of the conditions set forth in the treaties, built large quantities of tanks and warplanes. We can read these effective increases in γ as leading to Messenian revolts on one hand, and to World War II on the other.

The defensive costs in unilateral or bilateral war (ρ and $\tilde{\rho}$) can reflect, in addition to the running costs attached to supplies and to the destruction of weapons during engagements, the large-scale destruction of civilian facilities. These were rare in Antiquity: indeed, although ancient texts relate the total destruction of numerous cities, archaeological analyzes reveal that this was generally not the case (Fachard & Harris 2021), and it is more of a literary *topos*, and the scale of the engagements was in fact often low. At the other extreme, the nuclear explosions at Hiroshima and Nagasaki in August 1945 or the fire-bombing of Tokyo in March of that year, for example, essentially leveled these cities at an immense economic cost, not to mention the hundreds of thousands of lives lost. We know based on the model that a higher gain from predation leads to more potential for war. On the other hand, given γ , an increasing cost of defensive or bilateral war reduces the incentive to go to war. Hence, the use of highly destructive weapons can lead to capitulation by making the cost of waging war unbearable to the enemy.

World War II carpet bombing or nuclear weapon use also are purely destructive and incompatible with economic predation of the facilities that are destroyed, and they mainly affect civilians. Their use is specifically aimed at deterring the enemy from continuing war (increasing ρ_{-i} and $\tilde{\rho}_{-i}$). Bilateral access to the most destructive weapons, such as thermonuclear

long-range missiles, may in fact lead to a peace equilibrium without requiring a high reluctance to human loss (v_f or v_d). In a “balance of terror” equilibrium, the potential cost of waging bilateral war inflicted on the enemy makes war undesirable. The use of such destructive weapons in bilateral conflict largely increases the enemy’s cost $\tilde{\rho}_{-i}$ and also reduces the ability to gain from predation (γ_i).

The ability to gain from war (γ_i), to inflict high costs to the enemy (ρ_{-i} and $\tilde{\rho}_{-i}$), and not suffer high costs oneself (ρ_i and $\tilde{\rho}_i$) depends on the nature of the offensive and defensive weapons available. These may or may not be transmitted from one generation to the next: this is driven by the parameter δ , which sets the attrition of capital. Defensive constructions, such as the “Cyclopean walls” of Mycenaean fortresses, Roman defensive bulwarks, or Medieval stone castles, some of them still standing, had a long life cycle and contributed to lower δ . On the other hand, for much of Antiquity, warriors were often buried with their arms, and these offensive weapons were therefore not transmitted to the next generation. As offensive weapons grew in complexity and in cost, they became more readily transferred to the next generation, reducing δ . Cannons, arquebuses, catapults, galleons or battleships had a long lifespan, longer than a generation (if they were not consumed or destroyed through their use in war). As technological advances accelerated, modern weapons began to become obsolete faster, and tanks, jet fighters or nuclear missiles are now operating for about a generation.

3.2 The Specific Cost of Human Life

We have related the impact of human life loss on the occurrence of war (parameters v_d and v_f) to each country’s political regime, and the potential disincentive afforded by this effect mechanically entails Kant’s perspective. Throughout history, the attitude of governing bodies towards war in this regard has nevertheless varied significantly, and the model’s parameterization can account for these different situations.

Waging war or going on campaigns and therefore suffering and causing human losses, in certain specific contexts, could improve an autocrat’s political stance: for example, the early Roman emperors Gaius (Caligula) and Claudius went to Gauls and to Britain for this very purpose, with no consideration for the human cost, either to their own armies or to the enemy’s (Alston 2007). Throughout Roman history, there are countless examples of generals being executed because they had not died on the battlefield, or had surrendered, which shows the low consideration granted to their own people: $v_d = v_f = 0$ in this case.

Greek Antiquity, on the other hand, gives us an interesting set of parameters in the context of the model, opposing the value of life and the democratic regime to some extent. Combat

in a Greek phalanx or rowing on a warship was an exercise in equality, each fighter playing a fundamentally similar role (Hanson [1991] 1994), an equality that has been associated with the emergence of democracy, and these fighters often stood alongside close friends or family members (Pitsoulis 2011). Actual combat was strenuous and required physical courage (Lazenby [1991] 1994). The lives of the fellow fighters were so important that they mattered significantly even if they were dead: for example, after the Battle of Arginusae where the Athenians were victorious, their generals were executed because they had failed to bring the dead back (Kagan [1987] 2013). However, the enemy's life had no value. In fact, it has been argued that the ancient Greeks could not experience post-traumatic stress disorder related to war, because killing enemies was perfectly in line with their set of values (Crowley 2014). This extreme dichotomy is also true in the case of a democratic regime such as Athens's in the Classical period. The city-state, as a democracy, was a major actor in wars, and initiated or took part in a large number of them (Pritchard 2010). For example, according to Thucydides, Athens attacked the city of Melos because the Melians did not want to take sides during the Peloponnesian War, slaughtered all the men and enslaved all women and children⁵. The parameters for ancient Athens would therefore be $v_d > 0$ and $v_f = 0$, and if we follow Crowley (2014), we may even consider $v_f < 0$.

In the Middle Ages, with what has been termed a “bellicose society” (Black [2002] 2005, 4), continuous improvements in offensive weapons led to large human losses. Engagements such as the Battle of Marignano (1515) between Swiss and French forces “were extremely costly in human life, since they were fought in a restricted area at relatively close quarters, imposed by the limited range of the weapons and the continued reliance on tactics which necessitated closing with the enemy” (Wilson [1999] 2005, 187). Indeed, “The military reflected the nature of society, and its actions exemplified current attitudes towards human life. Killing was generally accepted as necessary, both for civil society—against crime, heresy and disorder—and in international relations” (Black [2002] 2005, 4). While substantial effort had been put into improving armor and protection for knights in particular through the Middle Ages (DeVries 2006), the increased use of firearms led to much more widespread loss life (and increase in L), since traditional armors were not effective against arquebuses. From the 16th and 17th century, armors indeed became lighter, while at the same time broader conscription became possible thanks to the comparatively easier use of light firearms, in comparison with swords or bows. Hence, medieval or early modern conflicts between monarchical polities presumably reflected a high L , but also a lower v_d as had been the case, resulting in the flurry of conflicts that emerged in Europe, from the French-Italian wars from the late 15th century

⁵See Thucydides (1921) V, 84 sq.

to the Thirty Years War in the 17th Century.

With respect to the cost of human life, democracies in the 19th century differed from the militant democracies of the late 20th and early 21st centuries, but also from the regimes from the early modern period. A significant difference with the earlier case was that the capacity to create damage, and the scale of war-related destruction, had increased. As Gartner (2008) showed, leaders were quite aware of the likely extent of destruction that would be brought about by war between European countries. There were attempts to create a legal framework to precisely define war, in this context. At the end of the 19th century, there was an evolution towards the notion of shared values, laws of war that should be respected, and that once the enemy was not able to fight anymore, they should be protected. The embodiment of these values included the founding of the Red Cross in 1863, and the first Hague peace conference in 1899. If “the right of belligerents to adopt means of injuring the enemy is not unlimited”, these principles serve in fact to stress the limits that were set to them, as they were subordinated to military necessity (Freedman 2018, 26–31). Indeed, all belligerents resorted to chemical weapons, and gas in particular, in World War I. The greater occurrence of democratic regimes among the states in a position to wage war also implied a greater influence of popular feelings on the possibility of war, and, as a consequence, the realization that the civil population of the enemy state could be culpable in some way (Freedman 2018, 33).

More generally, to the extent that civilians are suspected to take up arms against an army, or act in favor of the enemy, they become potential targets. For example, in August 1914, after several French army divisions had taken Mulhouse, and were welcomed by the population, the German army forced them to retreat and targeted civilians because they had been sympathetic to the enemy. On a much larger scale, the Armenian genocide of 1914-1916 by the Turkish ruling party stemmed from the same logic. The distinction between the treatment granted to enemy fighters and to insurrectionists or civilian resisters evolved from the brutal colonial wars, where civilian populations were massacred as a way of weakening their military strength. General von Moltke, who had vanquished the French army at the Battle of Sedan in 1870, later wrote “all the sources of support for the hostile government must be considered, its finances, railroads, foodstuffs, even its prestige” (Freedman 2018, 39–40). In this context, the value attributed to enemy lives was very low, and much lower to that attributed to the lives of one’s fellow citizens, and we have, as in the ancient Greek case, $v_d > 0$ and $v_f = 0$.

While before World War I the citizen of the enemy state was seen as potentially aggressive, the distinction between combatant and non-combatant evolved towards one between military and civilian (Alexander 2007). Since a civilian workforce, although vulnerable, would be behind the war production effort of a belligerent, they may be targeted by the enemy, and

hence should be protected by law, now that air raids would become a possibility: this is why the 1923 Hague Draft Rules of Aerial Warfare addressed this situation. Law-abiding democracies would hence be expected to be mindful of their own citizens' lives, but also of those of the civilians in the enemy state. This type of rule only stressed the potential importance of civilians in the wars to come, more than providing any real protection; the bombing of Guernica on April 26th, 1937 illustrated the terrible efficiency of air raids, as well as their strategic use on civilian populations in order to terrorize the enemy.

For a long time, the Western perception of the USSR's evaluation of human life in war was that it was very low, with a high degree of acceptance of massive loss, which would be consistent with a non-democratic state. However, Sella ([1992] 2005) has shown that there was a substantial evolution from the late 1910s until 1990, under a nominally constant political setup: from the glorified death of a hero, to "fighting at all costs", to a more compassionate view of soldiers embodied in significant efforts geared towards military medical capabilities. Based on Sella's argument, v_d increased over time in this case. The lesser quality of military equipment that plagued the Red Army during World War II would translate into a higher ρ or $\tilde{\rho}$ (more economic cost in achieving a given military objective), but that was to a larger extent compensated for with the acceptance of greater human losses in terms of bilateral war decision, reflecting a low value for v_d .

In modern democracies, the greater importance allocated to lives lost has had a substantial impact on the willingness to go to war. As Gelpi, Feaver, & Reifler (2006) and Gelpi, Feaver, & Reifler (2009) showed, the casualties from the Vietnam War and from Iraq, both in terms of domestic and foreign deaths, have had a very strong impact on the willingness of the US to be at war, culminating with the withdrawal from Afghanistan.

As the potential for human loss increases due to technology, through more efficient offensive weapons, and the variable L increases, this deterrence further raises. As we have pointed out earlier, modern democracies have at times entered war, which would seem to go against the logic of Kant's perspective as well as that of the model. In the dyadic relationship between countries, we can nevertheless reflect a different value for L : a much larger value if one is considering war against another nuclear-equipped country, for example, and a lower value if nuclear weapons will not be used. Hence, the same democratic country may be at the same time essentially forbidden from going to war against certain countries, while potentially able to go to war against other countries depending on the conditions.

4 Conclusion

We have seen that the study of the relationships between war and democracy has a long history, and has involved developments mostly in peace studies and in political sciences, but also in macroeconomics. The macroeconomic model we developed in the vein of Solow allowed us to account for the predation gains and the cost of wars in a dyadic relationship between two countries, resulting in various long-run equilibria, depending on their relative sizes, capital stock, and economic or military effectiveness. In relation to the model from Lagerlöf (2010), we were able to qualify the steady-state equilibria in all generality, and even obtain some closed-form expressions in a few specific cases.

In order to account for the impact of political regimes, we introduced different degrees of aversion for human loss, and this effectively offered a formal proof of Kant's "theorem": the more democratic the country, the least likely it is to enter war. Even if the potential profits from predation increase to the point where perpetual war between autocracies would be inevitable according to the model, democracies will avoid war due to their stronger aversion to human loss. Furthermore, as the possibility for human loss increases, for example, through the use of nuclear weapons, so does this deterrence effect. As a result, the same democratic country may forbid itself from going to war against certain countries while theoretically able to go to war against other countries depending on such circumstances, which can explain proxy wars. The macrodynamic model can hence account for a broader range of stylized facts than the models from Lagerlöf (2010) and Hess & Orphanides (2001).

References

- Acemoglu, Daron, Mikhail Golosov, Aleh Tsyvinski, & Pierre Yared.** 2012. "A Dynamic Theory of Resource Wars." *The Quarterly Journal of Economics* 127 (1): 283–331.
- Acemoglu, Daron, & James A. Robinson.** 2005. *Economic Origins of Dictatorship and Democracy*. Cambridge: Cambridge University Press.
- Alesina, Alberto, Bryony Reich, & Alessandro Riboni.** 2020. "Nation-Building, Nationalism, and Wars." *Journal of Economic Growth* 25 (4): 381–430.
- Alexander, Amanda.** 2007. "The Genesis of the Civilian." *Leiden Journal of International Law* 20 (2): 359–376.
- Alston, Richard.** 2007. "The Military and Politics." In *The Cambridge History of Greek and Roman Warfare: Rome from the Late Republic to the Late Empire*, edited by Philip A. G. Sabin, Hans van Wees, & Michael Whitby, 2:176–197. Cambridge: Cambridge University Press.
- Appadurai, Arjun.** 2007. "Hope and Democracy." *Public Culture* 19 (1): 29–34.
- Babst, Dean V.** 1972. "Elective Governments: A Force for Peace." *Industrial Research* 4: 55–58.
- Besley, Timothy, & Masayuki Kudamatsu.** 2008. "Making Autocracy Work." In *Institutions and Economic Performance*, edited by Elhanan Helpman, 452–510. Cambridge: Harvard University Press.
- Black, Jeremy.** (2002) 2005. *European Warfare, 1494-1660*. Warfare and History. London: Routledge.
- Boix, Carles.** 2003. *Democracy and Redistribution*. Cambridge: Cambridge University Press.
- Bremer, Stuart A.** 1993. "Democracy and Militarized Interstate Conflict, 1816–1965." *International Interactions* 18 (3): 231–249.
- Bueno de Mesquita, Bruce, & David Lalman.** 1992. *War and Reason: Domestic and International Imperatives*. New Haven: Yale University Press.
- Burger, Gerhard, Michael Grayling J., & Tomás Capretto.** 2022. "phaseR." phaseR. 2022. <https://github.com/mjg211/phaseR>.
- Carroll, Stuart.** 2017. "Thinking with Violence." *History and Theory* 56 (4): 23–43.
- Carter, Jeff.** 2017. "The Political Cost of War Mobilization in Democracies and Dictatorships." *Journal of Conflict Resolution* 61 (8): 1768–1794.
- Carter, Jeff, Scott Wolford, & D. Scott Bennett.** 2022. "Correlates of War." Correlates of War. 2022. <https://correlatesofwar.org>.
- Cartledge, Paul.** 2002. *Sparta and Lakonia: A Regional History, 1300-362 B.C.* 2nd ed. New York: Routledge.
- Clausewitz, Carl von.** (1832) 1989. *On War*. Translated by Michael Howard & Peter Paret. Princeton: Princeton University Press.

- Colaresi, Michael, & Sabine C. Carey.** 2008. "To Kill or to Protect: Security Forces, Domestic Institutions, and Genocide." *Journal of Conflict Resolution* 52 (1): 39–67.
- Collier, Paul, & Anke Hoeffler.** 2002. "Aid, Policy and Peace: Reducing the Risks of Civil Conflict." *Defence and Peace Economics* 13 (6): 435–450.
- Crowley, Jason.** 2014. "Beyond the Universal Soldier: Combat Trauma in Classical Antiquity." In *Combat Trauma and the Ancient Greeks*, edited by Peter Meineck & David Konstan, 105–130. New York: Palgrave Macmillan.
- Dahl, Robert A.** (1971) 2007. *Polyarchy: Participation and Opposition*. New Haven: Yale University Press.
- Davenport, Christian.** 2007. *State Repression and the Domestic Democratic Peace*. Cambridge: Cambridge University Press.
- DeVries, Kelly.** 1992. *Medieval Military Technology*. Peterborough: Broadview Press.
- . 2006. "Medieval Warfare and the Value of a Human Life." In *Noble Ideals and Bloody Realities: Warfare in the Middle Ages*, edited by Niall Christie & Maya Yazigi, 27–55. History of Warfare. Leiden: Brill.
- Dixon, William J.** 1993. "Democracy and the Management of International Conflict." *Journal of Conflict Resolution* 37 (1): 42–68.
- . 1994. "Democracy and the Peaceful Settlement of International Conflict." *American Political Science Review* 88 (1): 14–32.
- Doyle, Michael W.** 1986. "Liberalism and World Politics." *American Political Science Review* 80 (4): 1151–1169.
- Eck, Kristine, & Lisa Hultman.** 2007. "One-Sided Violence Against Civilians in War: Insights from New Fatality Data." *Journal of Peace Research* 44 (2): 233–246.
- Eckhardt, William.** 1991. "War-Related Deaths Since 3000 BC." *Bulletin of Peace Proposals* 22 (4): 437–443.
- Elbadawi, Ibrahim, & Nicholas Sambanis.** 2002. "How Much War Will We See?: Explaining the Prevalence of Civil War." *Journal of Conflict Resolution* 46 (3): 307–334.
- Fachard, Sylvian, & Edward Monroe Harris, eds.** 2021. *The Destruction of Cities in the Ancient Greek World: Integrating the Archaeological and Literary Evidence*. Cambridge: Cambridge University Press.
- Farber, Henry S., & Joanne Gowa.** 1995. "Polities and Peace." *International Security* 20 (2): 123.
- . 1997. "Common Interests or Common Polities? Reinterpreting the Democratic Peace." *The Journal of Politics* 59 (2): 393–417.
- Fleck, Robert K., & F. Andrew Hanssen.** 2006. "The Origins of Democracy: A Model with Application to Ancient Greece." *The Journal of Law and Economics* 49 (1): 115–146.

- Fleck, Robert K., & F. Andrew Hanssen.** 2013. "How Tyranny Paved the Way to Democracy: The Democratic Transition in Ancient Greece." *The Journal of Law and Economics* 56 (2): 389–416.
- Freedman, Lawrence.** 2018. *The Future of War: A History*. History. London: Penguin.
- Freedom House.** 2022. "Freedom in the World Research Methodology." Freedom House. 2022. <https://freedomhouse.org/reports/freedom-world/freedom-world-research-methodology>.
- Galor, Oded, & David N. Weil.** 2000. "Population, Technology, and Growth: From Malthusian Stagnation to the Demographic Transition and Beyond." *American Economic Review* 90 (4): 806–828.
- Gartner, Scott Sigmund.** 2008. "The Multiple Effects of Casualties on Public Support for War: An Experimental Approach." *American Political Science Review* 102 (1): 95–106.
- Gartzke, Erik.** 2007. "The Capitalist Peace" 51 (1): 166–191.
- Gates, Scott, Torbjørn L. Knutsen, & Jonathon W. Moses.** 1996. "Democracy and Peace: A More Skeptical View." *Journal of Peace Research* 33 (1): 1–10.
- Gelpi, Christopher, Peter D. Feaver, & Jason Reifler.** 2006. "Success Matters: Casualty Sensitivity and the War in Iraq." *International Security* 30 (3): 7–46.
- . 2009. *Paying the Human Costs of War: American Public Opinion and Casualties in Military Conflicts*. Princeton: Princeton University Press.
- Gibler, Douglas M.** 2007. "Bordering on Peace: Democracy, Territorial Issues, and Conflict." *International Studies Quarterly* 51 (3): 509–532.
- Gibler, Douglas M., Steven V. Miller, & Erin K. Little.** 2016. "An Analysis of the Militarized Interstate Dispute (MID) Dataset, 1816–2001." *International Studies Quarterly* 60 (4): 719–730.
- Gleditsch, Kristian S., & Michael D. Ward.** 1999. "A Revised List of Independent States Since the Congress of Vienna." *International Interactions* 25 (4): 393–413.
- Gleditsch, Nils P.** 1992. "Democracy and Peace." *Journal of Peace Research* 29 (4): 369–376.
- Gleditsch, Nils P., & Håvard Hegre.** 1997. "Peace and Democracy: Three Levels of Analysis." *Journal of Conflict Resolution* 41 (2): 283–310.
- Gleditsch, Nils P., Håvard Hegre, & Håvard Strand.** 2009. "Democracy and Civil War." In *Handbook of War Studies*, edited by Manus Midlarsky, 3:155–192. Ann Arbor: University of Michigan Press.
- Goenner, Cullen F.** 2004. "Uncertainty of the Liberal Peace." *Journal of Peace Research* 41 (5): 589–605.
- . 2007. "Economic War and Democratic Peace." *Conflict Management and Peace Science* 24 (3): 171–182.

- Grayling, Michael, J.** 2014. "phaseR: An R Package for Phase Plane Analysis of Autonomous ODE Systems." *The R Journal* 6 (2): 43.
- Gurr, Ted R.** 1974. "Persistence and Change in Political Systems, 1800–1971." *American Political Science Review* 68 (4): 1482–1504.
- , ed. 1978. *Comparative Studies of Political Conflict and Change, Cross-National Datasets*. Ann Arbor: Inter-university Consortium for Political and Social Research.
- Gurr, Ted R., Keith Jagers, & Will H. Moore.** 1990. "The Transformation of the Western State: The Growth of Democracy, Autocracy, and State Power Since 1800." *Studies In Comparative International Development* 25 (1): 73–108.
- Hadenius, Axel, & Jan. Teorell.** 2007. "Pathways from Authoritarianism." *Journal of Democracy* 18 (1): 143–157.
- Hansen, Gary D., & Edward C. Prescott.** 2002. "Malthus to Solow." *American Economic Review* 92 (4): 1205–1217.
- Hanson, Victor Davis.** (1991) 1994. "Hoplite Technology in Phalanx Battle." In *Hoplites: The Classical Greek Battle Experience*, edited by Victor Davis Hanson, 63–84. London: Routledge.
- Hegre, Håvard.** 2014. "Democracy and Armed Conflict." *Journal of Peace Research* 51 (2): 159–172.
- Hess, Gregory D., & Athanasios Orphanides.** 2001. "War and Democracy." *Journal of Political Economy* 109 (4): 776–810.
- Jackson, Matthew O., & Massimo Morelli.** 2007. "Political Bias and War." *American Economic Review* 97 (4): 1353–1373.
- Jagers, Keith, & Ted R. Gurr.** 1995. "Tracking Democracy's Third Wave with the Polity III Data." *Journal of Peace Research* 32 (4): 469–482.
- Kagan, Donald.** (1987) 2013. "The Trial of the Generals." In *New History of the Peloponnesian War*, 374–395. Ithaca: Cornell University Press.
- Kant, Immanuel.** (1795) 1991. *Political Writings*. Edited by Hans Siegbert Reiss. Translated by Hugh B. Nisbet. 2nd ed. Cambridge Texts in the History of Political Thought. Cambridge: Cambridge University Press.
- Keenan-Jones, Duncan, & Mark Hebblewhite.** 2019. "The Pitfalls of Using Ancient Population, Army and Casualty Data Without Expert Curation: A Review of Oka Et Al. 2017." *Cliodynamics* 10 (1): 54–66.
- Kress, Moshe.** 2020. "Lanchester Models for Irregular Warfare." *Mathematics* 8 (5): 737.
- Lacina, Bethany.** 2006. "Explaining the Severity of Civil Wars." *Journal of Conflict Resolution* 50 (2): 276–289.

- Lagerlöf, Nils-Petter.** 2010. "From Malthusian War to Solovian Peace." *Review of Economic Dynamics* 13 (3): 616–636.
- Lanchester, Frederick W.** 1916. *Aircraft in Warfare: The Dawn of the Fourth Arm*. London: Constable and Company Limited.
- . 1956. "Mathematics in Warfare." In *The World of Mathematics*, edited by James R. Newman, 4:2138–2157. New York: Simon & Schuster.
- Lazenby, John.** (1991) 1994. "The Killing Zone." In *Hoplites: The Classical Greek Battle Experience*, edited by Victor Davis Hanson, 87–109. London: Routledge.
- Le Fur, Tanguy, & Étienne Wasmer.** 2023. "The Multiplier Effect of Resource Conflicts: The Great Divergence and Reconvergence Through the Lens of Unified Growth Theory." Working Paper. Lille: Université de Lille.
- Levy, Jack.** 1988. "Domestic Politics and War." *Journal of Interdisciplinary History* 18 (4): 653–673.
- Lucas, Thomas W., & John A. Dinges.** 2004. "The Effect of Battle Circumstances on Fitting Lanchester Equations to the Battle of Kursk." *Military Operations Research* 9 (2): 17–30.
- Mansfield, Edward D., & Jack Snyder.** 1995. "Democratization and War." *Foreign Affairs* 74 (3): 79–97.
- Maoz, Zeev, & Nasrin Abdolali.** 1989. "Regime Types and International Conflict, 1816-1976." *Journal of Conflict Resolution* 33 (1): 3–35.
- Mayer, Arno J.** 1969. "Internal Causes and Purposes of War in Europe, 1870-1956: A Research Assignment." *The Journal of Modern History* 41 (3): 292–303.
- Mousseau, Michael.** 2000. "Market Prosperity, Democratic Consolidation, and Democratic Peace." *Journal of Conflict Resolution* 44 (4): 472–507.
- Müller, Harald.** 2004. "The Antinomy of Democratic Peace." *International Politics* 41 (4): 494–520.
- Niall MacKay.** 2006. "Lanchester Combat Models." *Mathematics Today* 42 (5): 170–173.
- Oka, Rahul C., Marc Kissel, Mark Golitko, Susan Guise Sheridan, Nam C. Kim, & Agustín Fuentes.** 2017. "Population Is the Main Driver of War Group Size and Conflict Casualties." *Proceedings of the National Academy of Sciences* 114 (52): E11101–E11110.
- Olson, Mancur.** 1993. "Dictatorship, Democracy, and Development." *American Political Science Review* 87 (3): 567–576.
- Oneal, John R., & James Lee Ray.** 1997. "New Tests of the Democratic Peace: Controlling for Economic Interdependence, 1950-85." *Political Research Quarterly* 50 (4): 751–775.
- Oren, Ido.** 1995. "The Subjectivity of the 'Democratic' Peace: Changing U.S. Perceptions of Imperial Germany." *International Security* 20 (2): 147.

- Perry, Nigel.** 2011. "Applications of Historical Analyses in Combat Modelling." Canberra: Australian Government, Department of Defense.
- Pitsoulis, Athanassios.** 2011. "The Egalitarian Battlefield: Reflections on the Origins of Majority Rule in Archaic Greece." *European Journal of Political Economy* 27 (1): 87–103.
- Pritchard, David, ed.** 2010. *War, Democracy and Culture in Classical Athens*. Cambridge: Cambridge University Press.
- Raknerud, Arvid, & Håvard Hegre.** 1997. "The Hazard of War: Reassessing the Evidence for the Democratic Peace." *Journal of Peace Research* 34 (4): 385–404.
- Ray, James Lee.** 1993. "Wars Between Democracies: Rare, or Nonexistent?" *International Interactions* 18 (3): 251–276.
- . 1998. "Does Democracy Cause Peace?" *Annual Review of Political Science* 1: 27–46.
- Raymond, Gregory A.** 1994. "Democracies, Disputes, and Third-Party Intermediaries." *Journal of Conflict Resolution* 38 (1): 24–42.
- Reuter, Timothy.** 1985. "Plunder and Tribute in the Carolingian Empire." *Transactions of the Royal Historical Society* 35: 75–94.
- Richardson, Lewis F.** (1960) 1975. *Statistics of Deadly Quarrels*. Edited by Quincy Wright. Pacific Grove: Boxwood Press.
- Rosecrance, Richard N.** 1986. *The Rise of the Trading State: Commerce and Conquest in the Modern World*. New York: Basic Books.
- Rummel, Richard J.** 1975–1981. *Understanding Conflict and War*. 5 vols. New York: SAGE Publications, Inc.
- . (1997) 2017. *Power Kills: Democracy as a Method of Nonviolence*. Abingdon: Routledge.
- Russett, Bruce.** 1998. "A Neo-Kantian Perspective: Democracy, Interdependence and International Organizations in Building Security Communities." In *Security Communities*, edited by Emanuel Adler & Michael Barnett, 368–394. Cambridge: Cambridge University Press.
- Sella, Amnon.** (1992) 2005. *The Value of Human Life in Soviet Warfare*. London: Routledge.
- Sen, Amartya.** 1999. "Democracy as a Universal Value." *Journal of Democracy* 10 (3): 3–17.
- Sevastianova, Daria.** 2009. "Impact of War on Country Per Capita GDP: A Descriptive Analysis." *Peace Economics, Peace Science and Public Policy* 15 (1): 114–139.
- Singer, J. David.** 1972. "The 'Correlates of War' Project: Interim Report and Rationale." *World Politics* 24 (2): 243–270.
- Solow, Robert M.** 1956. "A Contribution to the Theory of Economic Growth." *The Quarterly Journal of Economics* 70 (1): 65.
- Thies, Clifford F., & Christopher F. Baum.** 2020. "The Effect of War on Economic Growth." *Cato Journal* 40 (1): 199–212.

- Thompson, William R.** 1996. "Democracy and Peace: Putting the Cart Before the Horse?" *International Organization* 50 (1): 141–174.
- Thucydides.** 1921. *History of the Peloponnesian War, Volume III, Books 5-6*. Translated by Charles F. Smith. Vol. 3. 4 vols. Loeb Classical Library 110. Cambridge: Harvard University Press.
- Töngür, Ünal, Sara Hsu, & Adem Yavuz Elveren.** 2015. "Military Expenditures and Political Regimes: Evidence from Global Data, 1963–2000." *Economic Modelling* 44: 68–79.
- Tridimas, George.** 2015. "War, Disenfranchisement and the Fall of the Ancient Athenian Democracy." *European Journal of Political Economy* 38: 102–117.
- Urban, William L.** 1998. "Victims of the Baltic Crusade." *Journal of Baltic Studies* 29 (3): 195–212.
- Vanhanen, Tatu.** 2000. "A New Dataset for Measuring Democracy, 1810-1998." *Journal of Peace Research* 37 (2): 251–265.
- Vreeland, James Raymond.** 2008. "The Effect of Political Regime on Civil War: Unpacking Anocracy." *Journal of Conflict Resolution* 52 (3): 401–425.
- Washburn, Alan, & Moshe Kress.** 2009. *Combat Modeling*. Vol. 134. International Series in Operations Research & Management Science. Boston: Springer.
- Wilson, Peter.** (1999) 2005. "European Warfare 1450-1815." In *War in the Early Modern World, 1450-1815*, edited by Jeremy Black, 177–206. London: Routledge.
- Wright, Quincy.** 1942a. *A Study of War*. Vol. 1. 2 vols. Chicago: University of Chicago Press.
- . 1942b. *A Study of War*. Vol. 2. 2 vols. Chicago: University of Chicago Press.

Mathematical Appendix

Based on the specifications laid out at the beginning of Section 2.1, the full macrodynamics, depending on the type of war being waged, are written as follows, reflecting the expressions for gains and costs.

In unilateral war, where Country 1 attacks Country 2:

$$\begin{cases} \frac{dk_{1,t}}{dt} = s_1 f_1(k_{1,t}) - \delta_1 k_{1,t} + \gamma_1 f_1(k_{1,t}) \wedge m_2 \frac{N_2}{N_1} f_2(k_{2,t}) - \rho_1 \frac{N_2}{N_1} f_2(k_{2,t}) \wedge m_1 f_1(k_{1,t}) \\ \frac{dk_{2,t}}{dt} = s_2 f_2(k_{2,t}) - \delta_2 k_{2,t} - \gamma_1 \frac{N_1}{N_2} f_1(k_{1,t}) \wedge m_2 f_2(k_{2,t}) \end{cases}$$

In the case of bilateral war, the equations are symmetrical:

$$\begin{cases} \frac{dk_{1,t}}{dt} = s_1 f_1(k_{1,t}) - \delta_1 k_{1,t} - (\rho_1 + \tilde{\rho}_1) \frac{N_2}{N_1} f_2(k_{2,t}) \wedge m_1 f_1(k_{1,t}) \\ \quad + \gamma_1 f_1(k_{1,t}) \wedge \left[\left(m_2 \frac{N_2}{N_1} f_2(k_{2,t}) - (\rho_2 + \tilde{\rho}_2) f_1(k_{1,t}) \right) \vee 0 \right] \\ \quad - \gamma_2 \frac{N_2}{N_1} f_2(k_{2,t}) \wedge \left[\left(m_1 f_1(k_{1,t}) - (\rho_1 + \tilde{\rho}_1) \frac{N_2}{N_1} f_2(k_{2,t}) \right) \vee 0 \right] \\ \frac{dk_{2,t}}{dt} = s_2 f_2(k_{2,t}) - \delta_2 k_{2,t} - (\rho_2 + \tilde{\rho}_2) \frac{N_1}{N_2} f_1(k_{1,t}) \wedge m_2 f_2(k_{2,t}) \\ \quad + \gamma_2 f_2(k_{2,t}) \wedge \left[\left(m_1 \frac{N_1}{N_2} f_1(k_{1,t}) - (\rho_1 + \tilde{\rho}_1) f_2(k_{2,t}) \right) \vee 0 \right] \\ \quad - \gamma_1 \frac{N_1}{N_2} f_1(k_{1,t}) \wedge \left[\left(m_2 f_2(k_{2,t}) - (\rho_2 + \tilde{\rho}_2) \frac{N_1}{N_2} f_1(k_{1,t}) \right) \vee 0 \right] \end{cases}$$

4.1 Proof of Steady-State Equilibria (Proposition 2.1)

The countries are identical, so $f_1 = f_2 = f$, $m_1 = m_2 = m$, $s_1 = s_2 = s$, $\delta_1 = \delta_2 = \delta$, $\gamma_1 = \gamma_2 = \gamma$, $\rho_1 = \rho_2 = \rho$ and $\tilde{\rho}_1 = \tilde{\rho}_2 = \rho$. If there is no war, we can directly write the steady states as in Solow (1956):

$$\begin{cases} k_1 = \frac{s}{\delta} f(k_1) \\ k_2 = \frac{s}{\delta} f(k_2) \end{cases}$$

With no war, the equilibrium capital k_n therefore verifies $k_n = \frac{s}{\delta} f(k_n)$. The output function f is assumed to be concave, to reflect diminishing marginal increments. As a result, the intersection of the curves of the form $y = k$ and $y = af(k) + b$ with $k > 0$, for $a > 0$ and b of any sign, is increasing as a function of a and b . We will use this remark in order to locate other steady-state equilibria relative to k_n or to each other.

If country 1 unilaterally attacks country 2, then the macroeconomic dynamics write:

$$\begin{cases} \frac{dk_{1,t}}{dt} = s f(k_{1,t}) - \delta k_{1,t} + \gamma f(k_{1,t}) \wedge m \frac{N_2}{N_1} f(k_{2,t}) - \rho \frac{N_2}{N_1} f(k_{2,t}) \wedge m f(k_{1,t}) \\ \frac{dk_{2,t}}{dt} = s f(k_{2,t}) - \delta k_{2,t} - \gamma \frac{N_1}{N_2} f(k_{1,t}) \wedge m f(k_{2,t}) \end{cases}$$

Hence, the steady-state in this case writes:

$$\begin{cases} 0 = sf(k_1) - \delta k_1 + \gamma f(k_1) \wedge m \frac{N_2}{N_1} f(k_2) - \rho \frac{N_2}{N_1} f(k_2) \wedge mf(k_1) \\ 0 = sf(k_2) - \delta k_2 - \gamma \frac{N_1}{N_2} f(k_1) \wedge mf(k_2) \end{cases}$$

In order to analyze the potential steady-states, we need to analyze the conditions under which $\gamma f(k_1) < m \frac{N_2}{N_1} f(k_2)$ and $\rho \frac{N_2}{N_1} f(k_2) < mf(k_1)$. We first assume that the parameters are such that these conditions are verified. The system then can be written:

$$\begin{cases} 0 = sf(k_1) - \delta k_1 + \gamma f(k_1) - \rho \frac{N_2}{N_1} f(k_2) \\ 0 = sf(k_2) - \delta k_2 - \gamma \frac{N_1}{N_2} f(k_1) \end{cases}$$

From this, we obtain, for the unilateral attacker u and the defender d , the steady-state values as a solution to a system:

$$\begin{cases} k_u = \frac{s+\gamma}{\delta} f(k_u) - \frac{\rho}{\delta} \frac{N_d}{N_u} f(k_d) \\ k_d = \frac{s}{\delta} f(k_d) - \frac{\gamma}{\delta} \frac{N_u}{N_d} f(k_u) \end{cases}$$

We can see that $\gamma N_u f(k_u) = s N_d f(k_d) - \delta N_d k_d$, and therefore $\gamma N_u f(k_u) < s N_d f(k_d) < m N_d f(k_d)$, if $m > s$. We also observe that $\rho N_d f(k_d) = N_u(s + \gamma) f(k_u) - \delta N_u k_u$. Hence, $\rho N_d f(k_d) < N_u(s + \gamma) f(k_u)$. If we assume that $m > s + \gamma$, then we have $\rho N_d f(k_d) < m N_u f(k_u)$. The condition $m > s + \gamma$ is hence sufficient for the steady-state to verify the assumptions we made earlier.

Further, since $k_d = \frac{s}{\delta} f(k_d) - \frac{\gamma}{\delta} \frac{N_u}{N_d} f(k_u) < \frac{s}{\delta} f(k_d)$, then $k_d < k_n$. Also, since $k_u = \frac{s+\gamma}{\delta} f(k_u) - \frac{\rho}{\delta} \frac{N_d}{N_u} f(k_d)$, then $k_u > k_n$ is equivalent to $\gamma f(k_u) > \rho \frac{N_d}{N_u} f(k_d)$, or $k_u > f^{-1} \left[\frac{\rho N_d}{\gamma N_u} f(k_d) \right]$. If the cost of war is very small, or if the attacker's population is very high relative to the defender, then this is generally verified. However, if $m < \rho$ (and as a consequence $\rho > \gamma + s$), then given the condition that $\gamma N_u f(k_u) < m N_d f(k_d)$, $k_u < k_n$ and unilateral war is pointless.

We now turn to the case where $\gamma f(k_1) < m \frac{N_2}{N_1} f(k_2)$ and $\rho \frac{N_2}{N_1} f(k_2) > mf(k_1)$. In this case, the attacker's production is relatively low compared to the costs of waging war. The system becomes:

$$\begin{cases} 0 = sf(k_1) - \delta k_1 + \gamma f(k_1) - mf(k_1) \\ 0 = sf(k_2) - \delta k_2 - \gamma \frac{N_1}{N_2} f(k_1) \end{cases}$$

Hence, for the unilateral attacker u and the defender d :

$$\begin{cases} k_u = \frac{s+\gamma-m}{\delta} f(k_u) \\ k_d = \frac{s}{\delta} f(k_d) - \frac{\gamma}{\delta} \frac{N_u}{N_d} f(k_u) \end{cases}$$

For k_u to be non-zero, we need to assume that $m < \gamma + s$. As in the prior case, $\gamma N_u f(k_u) = s N_d f(k_d) - \delta N_d k_d$ and therefore $\gamma N_u f(k_u) < m N_d f(k_d)$. Note that for k_u to be greater than k_n , one needs to have $\gamma > m$. Also, from the expression for k_d , we can derive: $\rho N_d f(k_d) = \frac{\rho \gamma}{s} N_u f(k_u) + \frac{\rho \delta}{s} N_d k_d$. Therefore, $\rho N_d f(k_d) > \frac{\rho \gamma}{s} N_u f(k_u)$, and if $m < \frac{\rho \gamma}{s}$, then the condition $\rho N_d f(k_d) > m N_u f(k_u)$ is verified. If the maximum rate of expenses or predation m is low enough, then this particular steady-state can exist. Finally, we can see that $k_d < k_n$, and assuming $\gamma > m$, $k_u > k_d$.

In the two other cases, where $\gamma N_1 f(k_1) > m N_2 f(k_2)$, the expressions with γ disappear from the steady-state equations, and it is not possible to determine general conditions on the parameters so that the assumed conditions are verified. In both cases, indeed, the equation for k_2 only depends on k_2 .

When the attacker's production reaches above a certain level relative to the defender's, and both conditions $\gamma N_u f(k_u) > m N_d f(k_d)$ and $m N_u f(k_u) > \rho N_d f(k_d)$ are verified, then there is a new steady-state:

$$\begin{cases} k_u = \frac{s}{\delta} f(k_u) + \frac{N_d}{N_u} \frac{m-\rho}{\delta} f(k_d) \\ k_d = \frac{s-m}{\delta} f(k_d) \end{cases}$$

This steady-state is different from zero for the defender only if $m < s$, and we have $k_d < k_n$. If $m > \rho$, then $k_u > k_n$.

If both countries symmetrically go to war, then the dynamics becomes:

$$\begin{cases} \frac{dk_{1,t}}{dt} = s f(k_{1,t}) - \delta k_{1,t} - (\rho + \tilde{\rho}) \frac{N_2}{N_1} f(k_{2,t}) \wedge m f(k_{1,t}) \\ \quad + \gamma f(k_{1,t}) \wedge \left[\left(m \frac{N_2}{N_1} f(k_{2,t}) - (\rho + \tilde{\rho}) f(k_{1,t}) \right) \vee 0 \right] \\ \quad - \gamma \frac{N_2}{N_1} f(k_{2,t}) \wedge \left[\left(m f(k_{1,t}) - (\rho + \tilde{\rho}) \frac{N_2}{N_1} f(k_{2,t}) \right) \vee 0 \right] \\ \frac{dk_{2,t}}{dt} = s f(k_{2,t}) - \delta k_{2,t} - (\rho + \tilde{\rho}) \frac{N_1}{N_2} f(k_{1,t}) \wedge m f(k_{2,t}) \\ \quad + \gamma f(k_{2,t}) \wedge \left[\left(m \frac{N_1}{N_2} f(k_{1,t}) - (\rho + \tilde{\rho}) f(k_{2,t}) \right) \vee 0 \right] \\ \quad - \gamma \frac{N_1}{N_2} f(k_{1,t}) \wedge \left[\left(m f(k_{2,t}) - (\rho + \tilde{\rho}) \frac{N_1}{N_2} f(k_{1,t}) \right) \vee 0 \right] \end{cases}$$

Since the situation is symmetrical for both countries, let us make the assumption that production is equal for both countries at the steady-state: $N_1 f(k_1) = N_2 f(k_2)$. With that assumption, the predation that each country suffers and itself exerts on the other cancel out, and the steady-state conditions are simplified. We can write, for Country 1:

$$0 = s f(k_1) - \delta k_1 - [(\rho + \tilde{\rho}) \wedge m] f(k_1).$$

The same expression can be written for Country 2, so that we can define a capital stock

steady-state level k_b for both countries, such that $k_b = \frac{s-(\rho+\tilde{\rho})\wedge m}{\delta} f(k_b)$. If $\rho + \tilde{\rho} < m$, then this reduces to $k_b = \frac{s-\rho-\tilde{\rho}}{\delta} f(k_b)$, which is positive if $s - \rho - \tilde{\rho} > 0$. If $\rho + \tilde{\rho} > m$, then the steady-state verifies $k_b = \frac{s-m}{\delta} f(k_b)$, which is positive if $s > m$. In both cases, we can observe that $k_b < k_n$.

Finally, note that in the case of unilateral or bilateral war and assuming $f(0) = 0$, in the case of a capitulation where $k_{-i} = 0$, then the dynamics result in $k_i = k_n$.

4.2 Proof of the Number and Existence of Steady-States (Proposition 2.2)

Equations of the form $k = af(k)$ for $a > 0$ have a unique solution that is non zero, provided that $f'(0) > \frac{1}{a}$. With our assumptions on f , this is verified for our choice of parameters. The steady-state equations for peace or bilateral war therefore have a single non-trivial solution.

In the case of unilateral war, and with the assumptions of the proposition, the steady-state equations for Country 1 attacking Country 2 write:

$$\begin{cases} 0 = sf(k_1) - \delta k_1 + \gamma f(k_1) - \rho f(k_2) \\ 0 = sf(k_2) - \delta k_2 - \gamma f(k_1) \end{cases}$$

We are assuming that k_2 is not small enough relative to k_1 that $\gamma f(k_1) > mf(k_2)$, given that m is large. Rearranging we obtain:

$$\begin{cases} k_2 = f^{-1} \left[\frac{s+\gamma}{\rho} f(k_1) - \frac{\delta}{\rho} k_1 \right] \\ k_1 = f^{-1} \left[\frac{s}{\gamma} f(k_2) - \frac{\delta}{\gamma} k_2 \right] \end{cases}$$

Solving for k_1 through substitution, we get:

$$\begin{aligned} f(k_1) &= \frac{s}{\gamma} \left[\frac{s+\gamma}{\rho} f(k_1) - \frac{\delta}{\rho} k_1 \right] - \frac{\delta}{\gamma} f^{-1} \left[\frac{s+\gamma}{\rho} f(k_1) - \frac{\delta}{\rho} k_1 \right] \\ \frac{s(s+\gamma) - \gamma\rho}{\delta\rho} f(k_1) - \frac{s}{\rho} k_1 &= f^{-1} \left[\frac{s+\gamma}{\rho} f(k_1) - \frac{\delta}{\rho} k_1 \right] \\ Z(k_1) &= X(k_1), \end{aligned}$$

where we define $Z(x) = \frac{s(s+\gamma) - \gamma\rho}{\delta\rho} f(x) - \frac{s}{\rho} x$, and $X(x) = f^{-1} \left[\frac{s+\gamma}{\rho} f(x) - \frac{\delta}{\rho} x \right] = f^{-1}(h(x))$. Excluding the case where $\delta = 0$, and the trivial cases where $k_1 = 0$, we can establish that (1) the equations $Z(x) = 0$ and $X(x) = 0$ have unique solutions:

- The equation $Z(x) = 0$ has a unique solution \check{k}_1^Z verifying $\frac{f(\check{k}_1^Z)}{\check{k}_1^Z} = \frac{s\delta}{s(s+\gamma) - \gamma\rho}$, assuming $f'(0) > \frac{s\delta}{s(s+\gamma) - \gamma\rho}$,
- The equation $X(x) = 0$ has a unique solution \check{k}_1^X verifying $\frac{f(\check{k}_1^X)}{\check{k}_1^X} = \frac{\delta}{s+\gamma}$, assuming $f'(0) > \frac{\delta}{s+\gamma}$.

Further, assuming that f is twice derivable on the domain of interest, we can establish that (2) the functions Z and X each have a unique maximum, because:

- $Z'(x) = 0$ at a unique value $\hat{k}_1^Z = f'^{-1}\left(\frac{s\delta}{\rho(s+\gamma)\left(\frac{s}{\rho}(s+\gamma)-\gamma\right)}\right)$,
- $X'(x) = \frac{s+\gamma}{\rho} f'(x) f'^{-1}(h(x)) = 0$ at a unique value $\hat{k}_1^X = f'^{-1}\left(\frac{\delta}{s+\gamma}\right)$.

Finally, (3) the functions Z' and X' are monotonically decreasing:

- $Z''(x) = \left(\frac{s(s+\gamma)}{\delta\rho} - \frac{\gamma}{\delta}\right) f''(x)$, so $Z''(x)$ is negative since $s > \rho$ implies that $s(s+\gamma) > \gamma\rho$,
- $X''(x) = \frac{s+\gamma}{\rho} f''(x) f'^{-1}(h(x)) + \left(\frac{s+\gamma}{\rho} f'(x)\right)^2 f''^{-1}(h(x))$ is always negative.

From points (1) and (2) above, we can infer that $\hat{k}_1^X > \hat{k}_1^Z$ is equivalent to $\check{k}_1^X > \check{k}_1^Z$. From all points (1), (2) and (3), we can infer the distinct conditions for 0, 1 or 2 non trivial solutions to the steady-state equation $Z(k_1) = X(k_1)$. Note that the condition $\check{k}_1^X > \check{k}_1^Z$ is equivalent to:

$$\frac{s\delta}{s(s+\gamma) - \gamma\rho} < \frac{\delta}{s+\gamma}$$

$$\frac{s}{\rho}(s+\gamma) - \gamma < 1.$$

We hence have the following:

- Conditions for 0 non-trivial solutions: $\check{k}_1^X - \check{k}_1^Z$ has the same sign as $X(\hat{k}_1^X) - Z(\hat{k}_1^Z)$;
- Conditions for 1 non-trivial solution: $\check{k}_1^X - \check{k}_1^Z$ does not have the same sign as $X(\hat{k}_1^X) - Z(\hat{k}_1^Z)$, and has the same sign as $Z'(0) - X'(0)$;
- Conditions for 2 non-trivial solutions: $\check{k}_1^X - \check{k}_1^Z$ does not have the same sign as $X(\hat{k}_1^X) - Z(\hat{k}_1^Z)$, and does not have the same sign as $Z'(0) - X'(0)$.

Hence we can see that the value of $\frac{s}{\rho}(s+\gamma) - \gamma$ will affect the number of steady-states.

4.3 Proof of the Local Stability of Steady-States (Proposition 2.3)

We denote $x_{i,t} = k_{i,t} - k_i$ where k_i is a steady state.

In the neighborhood of a perpetual peace steady-state, the local dynamical system evolves as follows:

$$\begin{bmatrix} \frac{dx_{1,t}}{dt} \\ \frac{dx_{2,t}}{dt} \end{bmatrix} = \begin{bmatrix} sf'(k_1) - \delta & 0 \\ 0 & sf'(k_2) - \delta \end{bmatrix} \begin{bmatrix} x_{1,t} \\ x_{2,t} \end{bmatrix}$$

$sf(k_i) - \delta k_i = 0$, from the concavity of f and $f(0) = 0$, we deduce that $sf'(k_i) - \delta < 0$. This condition is necessary and sufficient to guarantee stability.

In the neighborhood of a bilateral war steady-state, the local dynamical system evolves as follows:

$$\begin{bmatrix} \frac{dx_{1,t}}{dt} \\ \frac{dx_{2,t}}{dt} \end{bmatrix} = \begin{bmatrix} (s + \gamma) f'(k) - \delta & -(\gamma + \rho + \bar{\rho}) f'(k) \\ -(\gamma + \rho + \bar{\rho}) f'(k) & (s + \gamma) f'(k) - \delta \end{bmatrix} \begin{bmatrix} x_{1,t} \\ x_{2,t} \end{bmatrix}$$

Local stability is guaranteed if: $(s + \gamma) f'(k) - \delta < 0$ and

$$((s + \gamma) f'(k) - \delta)^2 > ((\gamma + \rho + \bar{\rho}) f'(k))^2.$$

These conditions are necessary and sufficient to guarantee the stability. If $(s + \gamma) f'(k) - \delta < 0$, then the second condition becomes $(s + \gamma) f'(k) - \delta < -(\gamma + \rho + \bar{\rho}) f'(k)$, which is equivalent to $(s + \gamma) f'(k) < \delta - (\gamma + \rho + \bar{\rho}) f'(k)$. When this condition is satisfied, then the first condition, $(s + \gamma) f'(k) < \delta$, is also always satisfied.

In the neighborhood of a unilateral war steady-state, the local dynamical system evolves as follows:

$$\begin{bmatrix} \frac{dx_{u,t}}{dt} \\ \frac{dx_{d,t}}{dt} \end{bmatrix} = \begin{bmatrix} (s + \gamma) f'(k_u) - \delta & -\rho f'(k_d) \\ -(\gamma + \bar{\rho}) f'(k_u) & s f'(k_d) - \delta \end{bmatrix} \begin{bmatrix} x_{u,t} \\ x_{d,t} \end{bmatrix}$$

Local stability is guaranteed if $(s + \gamma) f'(k_u) + s f'(k_d) - 2\delta < 0$ and

$$((s + \gamma) s - \rho(\gamma + \bar{\rho})) f'(k_u) f'(k_d) - \delta s f'(k_d) - \delta (s + \gamma) f'(k_u) + \delta^2 > 0.$$

This second condition can be rewritten as:

$$(s + \gamma) f'(k_u) + s f'(k_d) < \delta + \frac{((s + \gamma) s - \rho(\gamma + \bar{\rho}))}{\delta} f'(k_u) f'(k_d),$$

and this ends the proof.